Chapter 1

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Electric Charges & Fields

Electrostatics is study of electric charge at rest.

Electric Charge :-

Electric charge is an intrinsic property of elementary particles of matter which gives rise to electric force between various objects.

→ Transferable. $1\,mC = 10^{-3}\,C$

→ Scalar quantity. $1\,\mu C = 10^{-6}\,C$

→ SI unit is Coulomb (C).

→ Dimension [AT]

→ C.G.S unit is e.s.u ; $1C = 3 \times 10^{9}$ esu.

Types of charge -

1. Positive charge - If a body have deficiency of electrons on it, then charge developed on that body is called positive charge.

2. Negative charge - If a body have excess of electrons on it, then the body have negative charge on it.

Fundamental Law of Electrostatics :-

According to this law, Like charge repel and unlike charge attract each other.

$q_e = -1.6 \times 10^{-19}\,C$

$q_p = +1.6 \times 10^{-19}\,C$

$q_n = 0$

$m_e = 9.11 \times 10^{-31}\,Kg$

$m_p = 1.67 \times 10^{-27}\,Kg$

$m_n = 1.67 \times 10^{-27}\,Kg$

also, charge is always associated with mass. i.e Charge can't exist without mass.

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Classification of material with repect to charge :-

i). Conductor - Conductors are those material which allows charge to flow easily. They contain a large number of free electrons which make them good conductor of electricity.

eg. Metals, human and animal bodies, graphite acid, alkalies etc. are conductors.

ii). Insulator - Insulator are those material which do not allow electric charge to flow within it. Due to absence of free charge, these substance offer high resistance.

eg. Non-metals like glass, diamond, porcelain, plastic, nylon, wood, mica etc. are insulator.

iii). Semi-condutors - At low temperature it acts as insulator but at high temperature it acts as conductor.

eg. Silicon, Germanium etc.

Charging by Induction -

~~It~~ Most object are electrically neutral, which means that they have an equal numbers of positive and negative charges. In order to charge an object, one has to alter the charge balance of positive & negative charges.

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There are three ways to do it -

1). Charging by Friction -

The charging by friction involves rubbing of one particle on another resulting in electrons moving from one surface to another.

This method is useful for charging insulators.

2). Charging by conduction -

The charging by conduction process involves touching of a charged particle to a conductiv material (say +ve charged rod). It attract the negative charge and leaving the other sphere positively charged.

Now the sphere are separated before the rod is remove and thus separating positive and negative charge

Remove the rod . The charge on sphere will be rearranged and uniformly distributed over i

This method useful for charging conductors.

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3. Charging by induction :

The induction charging is a charging method that charges an object without actually touching object. The charging by induction process is where the charged particle is held near an uncharged conductive material which is grounded on a neutrally charged material. The charge flow between two object and uncharged conductive material develop a charge with opposite polarity. Polarising is necessary in this method.

Difference between charge and mass :-

	Charge	Mass.
1.	It can be positive or negative, zero.	1. It is always positive.
2.	It is always conserved.	2. It is not conserved by itself some mass change into energy.
3.	Charge is always associated with mass.	3. A body possessing charge mass may not have any net charge.
4.	charge does not depends on speed.	4. It depends on speed. Infact mass increased with speed.
5.	Charge is alway quantised.	5. Quantisation of mass not yet establised.
6.	Electrostatic force between two charge may be attractive or repulsive.	6. Gravitation between two masses always attractive.

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Basic Properties of electric charge:

charge have three basic properties -

→ Additivity
→ Quantisation
→ Conservation

• Additivity of charge means that the total charge of a system is algebraic sum of all individual charges located at different points inside a system.
If a system contain charge $q_1, q_2, \ldots q_n$ then total charge is -

$$\boxed{q = q_1 + q_2 + \cdots + q_n}$$

• Quantisation of electric charge means that the total charge (q) of a body is always an integral multiple of basic quantum of charge.

$$\boxed{Q = \pm n \cdot e}$$

where ;
Q = total charge of a body
n = ~~no~~ integer value
e = charge on one electron (1.6×10^{-19} C).

• Conservation of charge :
The law of conservation states that -
(i). The total charge of an isolated system remain conserved.
(ii). The electric charge can neither be created nor destroyed, they can be only transferred from one body to another.

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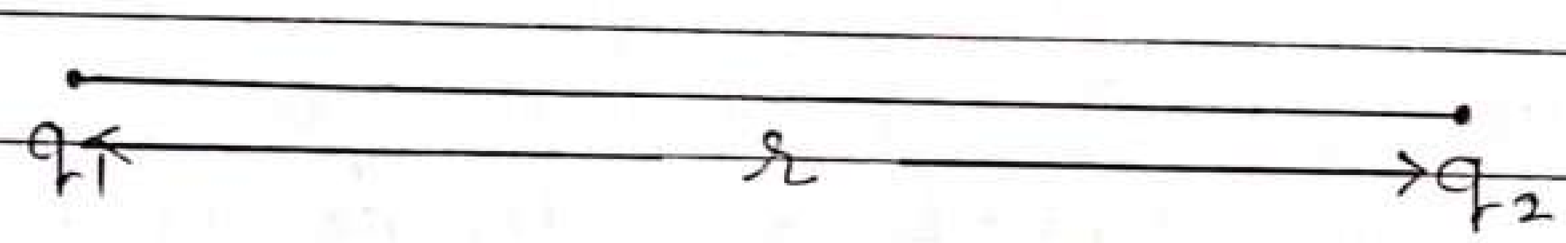
Example of conservation of charge :

When a glass rod is rubbed with a silk cloth it develops a positive charge. But at the same time, silk cloth develops equal negative charge. Thus, net charge before and after is zero.

Coulomb's Law :-

Coulomb's law states that force of attraction or repulsion between two stationary point charge is directly proportional to the product of the magnitude of two charges and inversely proportional to square of the distance between them. This force act along the line joining the two charges.



If two point charges q_1 and q_2, are separated by distance r, the force F of attraction or repulsion between them is such that

$$F \propto q_1 q_2 \quad \text{and} \quad F \propto \frac{1}{r^2}$$

$$\because \quad F \propto \frac{q_1 q_2}{r^2}$$

$$\boxed{F = \frac{k\, q_1 q_2}{r^2}}$$

also,

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \ Nm^2C^{-2}$$

where k is constant of proportionality, called Electrostatic force constant.

$$\boxed{F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}}$$, where ε° is permitivity of free space.

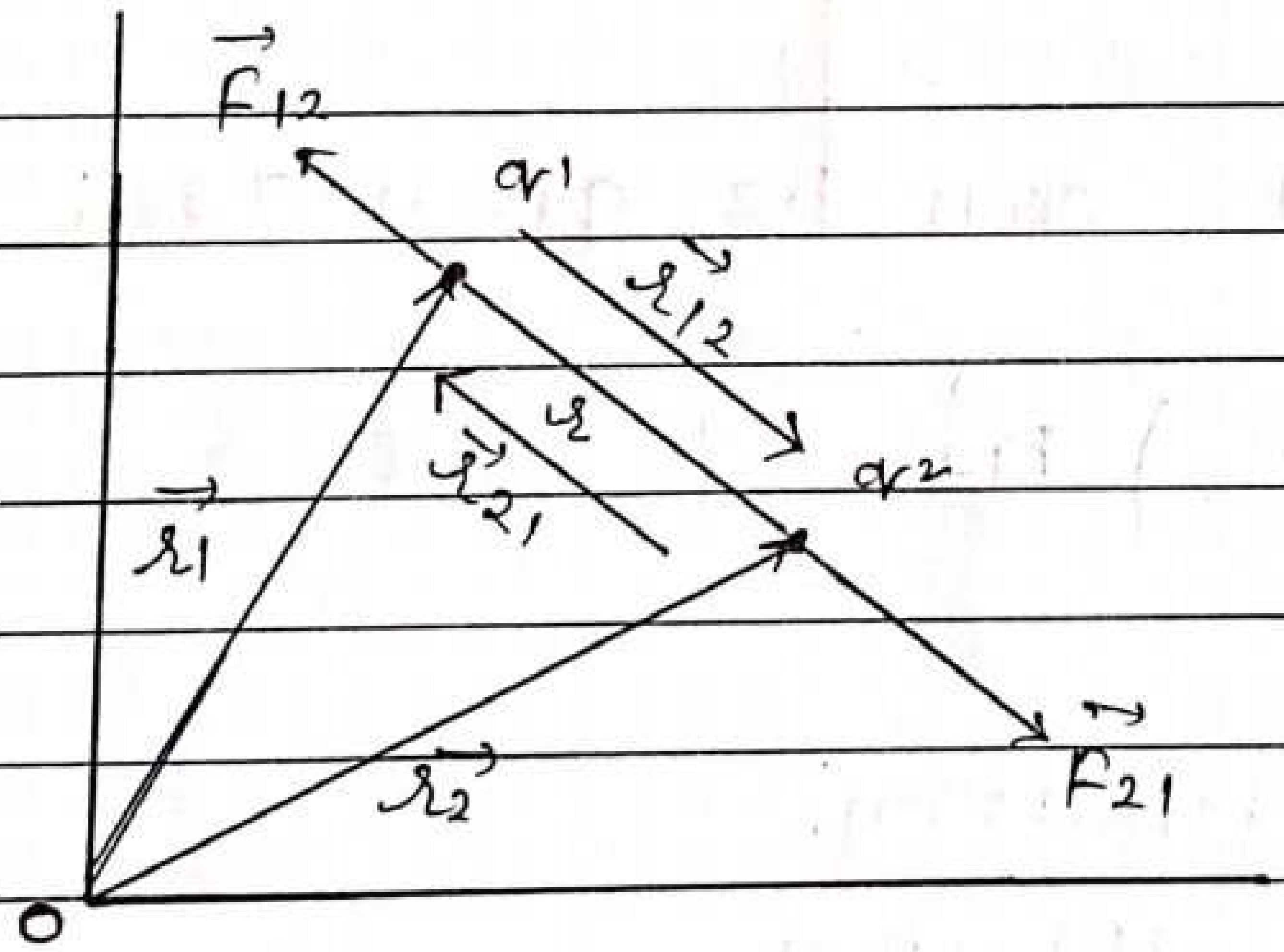
→ One coulomb is the amount of charge that repels equal and similar charge with a force of 9×10^9 N when placed in vaccum at 1 m distance between them.

K depends on nature of medium between two charges.

Dimension of K : $[ML^3T^{-4}A^{-2}]$
Unit of K : Nm^2C^{-2}

Dimension of permitivity (ε°) : $[M^{-1}L^{-3}T^4A^2]$
Unit of ε_0 = $N^{-1}m^{-2}C^2$

Coulomb's Law in vector form :-



Let the position vector of charges q_1 and q_2 be $\vec{r}_1$ and $\vec{r}_2$ respectively.
We denote the force on q_1 due to q_2 by $\vec{F}_{12}$ and the force on q_2 due to q_1 by $\vec{F}_{21}$

$\vec{F}_{21} = -\vec{F}_{12}$

Here, $\vec{r}_1 + \vec{r}_{12} = \vec{r}_2$ $\qquad$ $\vec{r}_2 + \vec{r}_{21} = \vec{r}_1$

$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$ also, $\vec{r}_{21} = \vec{r}_1 - \vec{r}_2$

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and $\vec{r_{12}} = -\vec{r_{21}}$

The magnitude of vector $\vec{r_{12}}$ and $\vec{r_{21}}$ is denoted by $|\vec{r_{12}}|$ and $|\vec{r_{21}}|$ and their unit vector is given as-

$$\hat{r_{12}} = \frac{\vec{r_{12}}}{|\vec{r_{12}}|} \quad \text{and} \quad \hat{r_{21}} = \frac{\vec{r_{21}}}{|\vec{r_{21}}|}$$

$\hat{A} = \frac{\vec{A}}{|A|}$

So, Force $\vec{F_{12}}$ can be given as ;

$$\vec{F_{12}} = \left(\frac{Kq_1q_2}{r_{21}^2}\right)\hat{r_{21}}$$

$$\vec{F_{12}} = \left(\frac{Kq_1q_2}{r_{21}^2}\right)\frac{\vec{r_{21}}}{|\vec{r_{21}}|}$$

$$\vec{F_{12}} = \left(\frac{Kq_1q_2}{|r_{21}|^3}\right)\vec{r_{21}}$$

Similarly, $\vec{F_{21}}$ can be given as ;

$$\vec{F_{21}} = \left(\frac{Kq_1q_2}{|r_{12}|^3}\right)\vec{r_{12}}$$

- $q_1q_2 > 0 \rightarrow$ repulsion.
- $q_1q_2 < 0 \rightarrow$ attraction

Importance of vector Form -

1. As $-\hat{r_{21}} = \hat{r_{12}}$, therefore $-\vec{F_{12}} = \vec{F_{21}}$ two charge exert equal and opposite force. So, Coulombian force obey Newton's 3rd law of motion.

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2. Coulombian force act line joining two charges, so they are central forces.

Limitation of coulomb's law :-

1.) The electric charge must be at rest.
2.) The electric charge must be point charges i.e. the extension of charges must be much smaller than separation between the charges.
3.) The separation between charge must be greater than the nuclear size (10^{-15} m) because strong nuclear force dominates electrostatic force

Dielectric Constant (K) :-

Force in vacuum $\Rightarrow F_0 = \frac{K q_1 q_2}{r^2}$

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \quad \text{--- (1)}$$

Force in medium $\Rightarrow F_m = \frac{K q_1 q_2}{r^2}$

$$F_m = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2} \quad \text{--- (2)}$$

Dividing (1) by (2), we get -

$$\frac{F_0}{F_m} = \frac{\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}}{\frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2}{r^2}} = \frac{\varepsilon}{\varepsilon_0}$$

ration of $(\varepsilon/\varepsilon_0)$ is called relative permitivity (ε_r) or dielectric constant (K) of given medium.

$$\varepsilon_0 = 8.85 \times 10^{-12}$$

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$$\boxed{\varepsilon_r \text{ or } K = \frac{\varepsilon}{\varepsilon_0} = \frac{F_0}{F_m}}$$

The dielectric constant or relative permitivity of a medium may be defined as the ratio of force between two charged species placed at some distance in free space to the force between same two charges when they are placed at same distance in a given medium.

$$F_m = \frac{F^\circ}{K}$$

$$F_m = \frac{1}{4\pi\varepsilon_0} \times \frac{1}{K} \times \frac{q_1 q_2}{r^2}$$

ε (Permitivity)
⇓
property of medium which determines the electric force between two charges placed in that medium.

$$\boxed{F_m = \frac{1}{4\pi\varepsilon_0 K} \cdot \frac{q_1 q_2}{r^2}}$$

$$\boxed{F_0 = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}}$$

K (air) = 1.00054
K (water) = 80

Similarity And Dissimilarity between F_E and F_G:-

→ Similarity
1). Both Forces obeys inverse square law.
2). Both proportional to product of charges and masses
3). Both act along line joining centers (central force).
4.). Conservation force.
5). Both forces can operate in vaccum.

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Dissimilarity -

1). F_E is attractive as well as repulsive but F_G is attractive only.

2). F_E depends on medium while F_G do not.

3). F_E are stronger than F_G.

Principle of Super-Position :-

According to this principle, when the number of charge are interacting, the total force on a given charge is the vector sum of forces exerted on it due to all other charges. The force between two charge is not affected by the presence of other charges.

Consider N point charges $q_1, q_2, q_3 --- q_N$ placed in vaccum at point whose position vector w.r.t. origin O are $\vec{r_1}, \vec{r_2}, \vec{r_3} --- \vec{r_N}$.

By superposition principle : $F_1 = \vec{F}_{12} + \vec{F}_{13} + -- \vec{F}_{1N}$

According to coulomb's law ;

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

So, total force on charge q_1 is -

$$\vec{F_1} = \frac{1}{4\pi\varepsilon_0} \left[\frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} + \frac{q_1 q_3}{r_{13}^2} \hat{r}_{13} + --- + \frac{q_1 q_N}{r_{1N}^2} \hat{r}_{1N} \right]$$

$$\boxed{\vec{F_1} = \frac{q_1}{4\pi\varepsilon_0} \sum_{i=2}^{N} \frac{q_i}{r_{1i}^2} \hat{r}_{1i}}$$

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In vector,

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \frac{\vec{r}_1 - \vec{r}_2}{|r_1 - r_2|^3}$$

So,

$$\boxed{\vec{F}_1 = \frac{q_1}{4\pi\varepsilon_0} \sum_{i=2}^{N} q_i \frac{\vec{r}_1 - \vec{r}_i}{|\vec{r}_1 - \vec{r}_i|^3}}$$

formula used for super position -

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + - - - + \vec{F}_{1N}$$

$$\boxed{F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}}$$

Electric Field :-

The region or space around a charge in which another charge experience electrostatic force of attraction or repulsion is known as electric field.

The extension of electric field of a charge is upto infinity. (∞).

Point charge : It is an electric charge considered to exis at a single point, and thus having neither area nor volume. ~~and hence no electric field of its own~~.

or

If the size of charged bodies are very small as compared to distance between them, then we treat it as point charge.

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Test charge : A test charge is a charge with a so small magnitude that placing it at a point has negligible affect on field around. It does not create any field of its own.

Electric Field intensity :- It is defined as the electrostatic force per unit test charge acting on vanishingly small positive test charge placed at given point.

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$

→ vector Quantity
→ its direction is in the direction of electrostatic force acting on positive charge.

→ SI Unit is NC^{-1} or Vm^{-1}.
→ Dimension is $[MLT^{-3}A^{-1}]$

▸ Electric Field due to a point charge q at a distance r from it is given by

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}}$$

Electric Field intensity depends on.

1). Magnitude of point charge.
2). Distance of point from charge.
3). Permitivity of medium.

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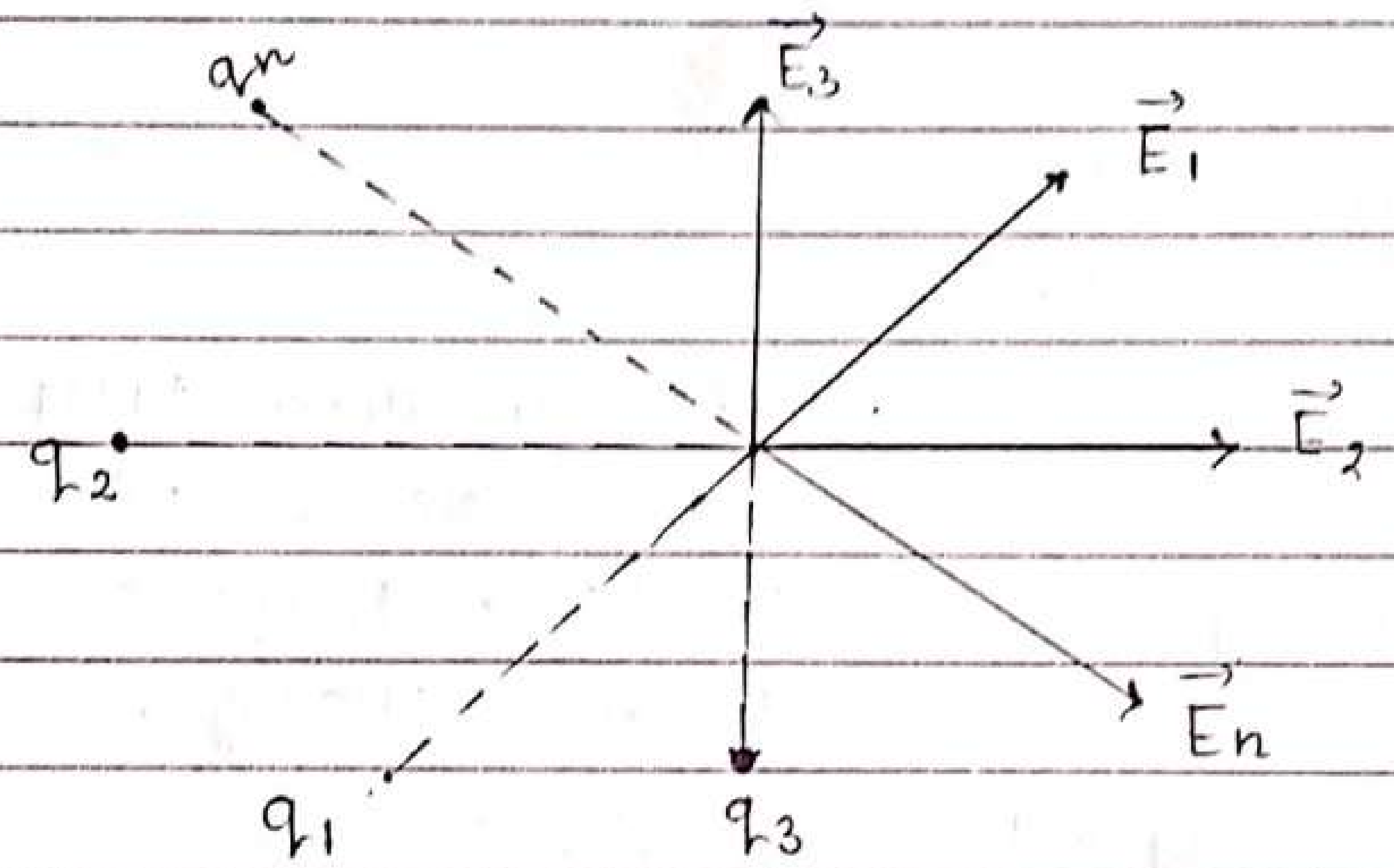
→ If q is positive, E points radially outwards and if q is negative E points, radially inwards.

Electric Field due to a system of point charges :

Consider a system of n charges q_1, q_2, q_3 ---- --- q_n due to which electric field is determined at a point P.



If r_1, r_2 ----- r_n is the distances of q_1, q_2, -- q_n charges from the point P and E_1, E_2 -- E_n is electric field du then,

$$\vec{E}_{net} = \vec{E_1} + \vec{E_2} + \vec{E_3}$$

$$= \boxed{\frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_{ip}^2} \hat{r}_{ip}}$$

or
$$= \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{|\vec{r} - \vec{r}_i|^3} (\vec{r} - \vec{r}_i)$$

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The principle states that the electric field at any point due to a group of point charge is equal to the vector sum of electric fields produced by each charge individually at that point, when all other charge assumed to be absent.

Continuous charge distribution :
whenever the charge is given to any conductor the charge spread all over the body uniformly.

Volume charge density , $\rho = \frac{dq}{dV}$ Cm^{-3}

Surface charge density , $\sigma = \frac{dq}{ds}$ Cm^{-2}

Linear charge density , $\lambda = \frac{dq}{dl}$ Cm^{-1}

variation of E with q and r

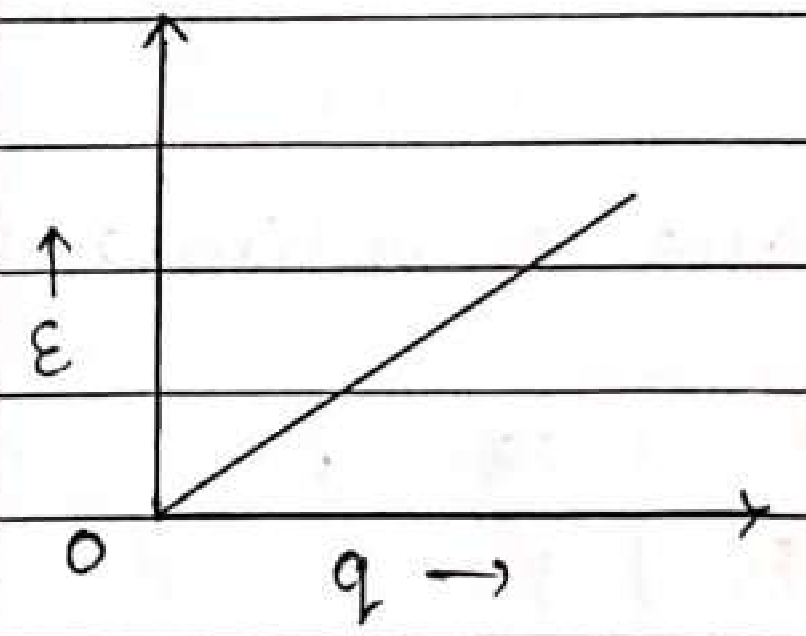
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

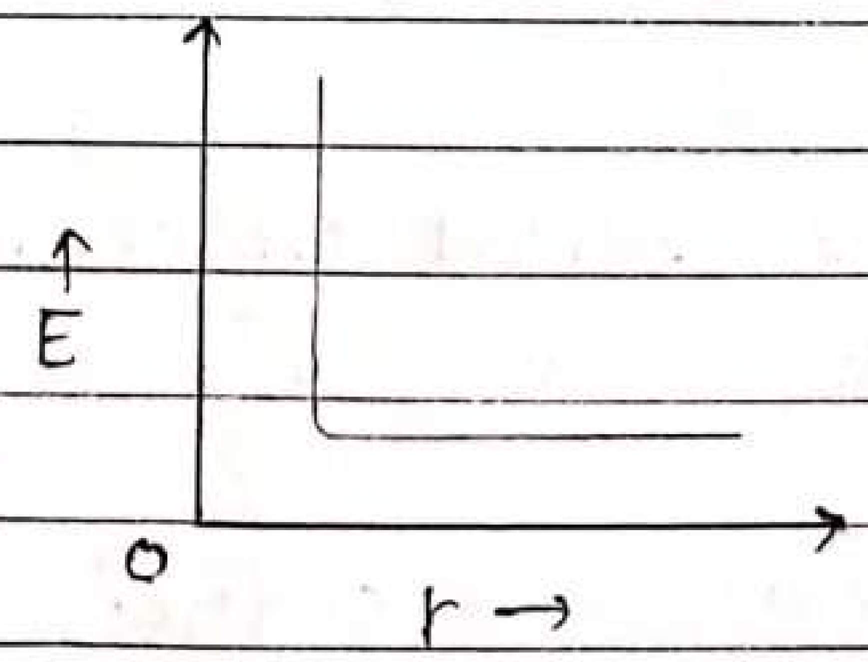
also,

$$E \propto q$$

$$E \propto \frac{1}{r^2}$$

graph :





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Electric field lines are the imaginary line in a region of space time along which a free (q_0) positive charge would move if allowed to do so in an electric field and tangent to which at any point gives direction of electric field at that point.

Properties of Electric field lines of force:

1). Lines of force are continuous curves without any breaks.

2). No two lines of force can cross each other.

3). They start at positive charges and ends at negative charge - they cannot form closed loops.

4). The relative closeness of the lines of force indicates the strength of electric field at different points.

5). They are always normal to the surface of the conductor.

6). They have a tendency to contract lengthwise and expand laterally.

Force exerted on charge q_0 due to continuous charge distribution,

$$\vec{F} = \frac{q_0}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\,\hat{r}$$

$\vec{E}$ due to continuous charge.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\,\hat{r}$$

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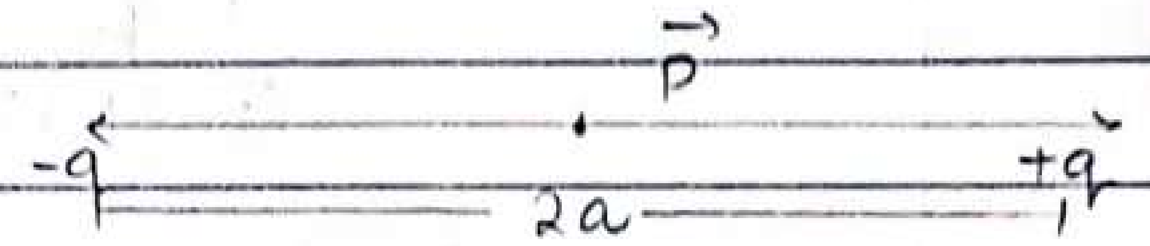
Electric Dipole :-

(+q and -q)
A pair of equal and opposite charges separated by a small distance (2a) is called electric dipole.

Electric Dipole movement :-

It measures the strength of electric dipole. The dipole movement of an electric dipole is vector whose magnitude is either charge times the separation between two opposite charge and direction is along the dipole axis from negative to positive charge.

$$\boxed{\vec{P} = q \times 2\vec{a}}$$



Dipole moment is a vector quantity having direction along dipole axis from -q to +q.
It's SI unit Coulomb meter (Cm).
Practiced unit ⇒ Debye.

Dipole Field :-

The electric field produced by an electric dipole is called a dipole field.

Note :- The total charge of an electric dipole is zero. But electric field of an electric dipole is not zero. This is because +q and -q are separated by some distance, so $\vec{E}$ field due to them when added do not exactly cancel out. However, at distance much larger than dipole size ($r >>> 2a$) the field nearly cancel out.

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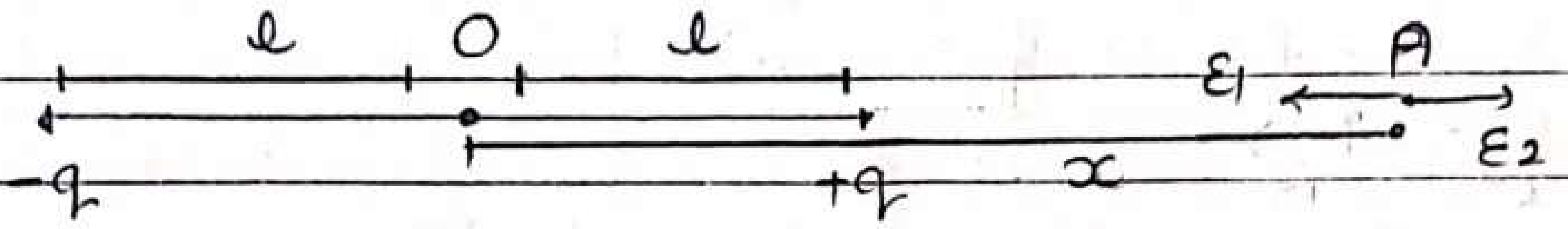
case (1) Electric field due to a dipole on axis of dipole at distance 'x' from centre of dipole :-



$$E_1 = \frac{kq}{r^2} = \frac{kq}{(x+l)^2}$$

$$E_2 = \frac{kq}{r^2} = \frac{kq}{(x-l)^2}$$

Net electric field $= E_2 - E_1$

$$= \frac{kq}{(x-l)^2} - \frac{kq}{(x+l)^2}$$

$$= kq\left[\frac{1}{(x-l)^2} - \frac{1}{(x+l)^2}\right]$$

$$= kq\left[\frac{(x+l)^2 - (x-l)^2}{((x-l)^2(x+l)^2)}\right]$$

$$= kq\left[\frac{4lx}{(x^2-l^2)^2}\right]$$

$$= \frac{2kq(2l)x}{(x^2-l^2)^2} \qquad \left[\because p = q \times 2l\right.$$

$$= \frac{2kpx}{(x^2-l^2)^2} \qquad \left[\because x >>> l \text{ ; neglect } l \quad l^2 \to 0\right]$$

$$= \frac{2kpx}{x^4}$$

$$\boxed{E_{net} = \frac{2kp}{x^3}} \qquad \boxed{\vec{E}_{axis} = \frac{2k\vec{p}}{x^3}}$$

→ $\vec{E}_{net}$ and $\vec{p}$ have same direction.

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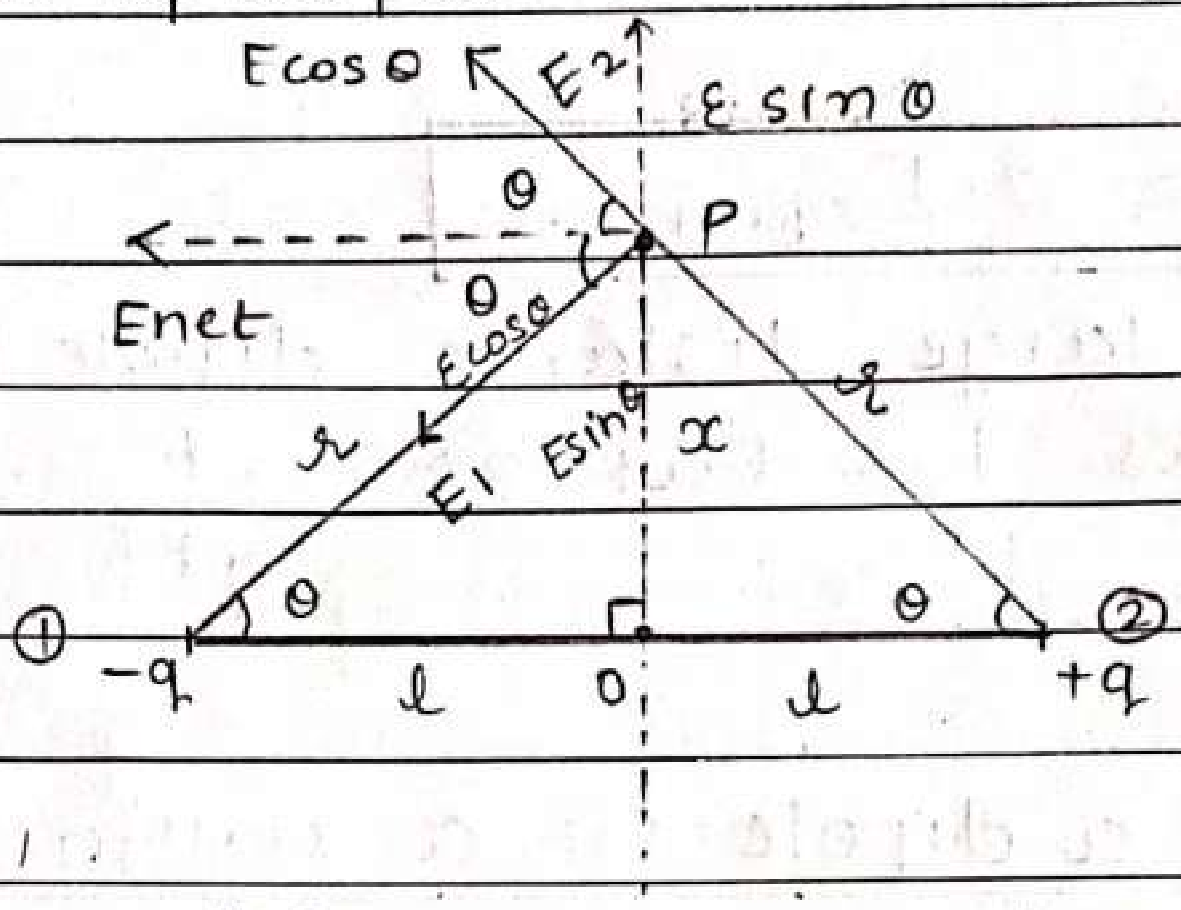
case ② Electric ~~dipole~~ field due to a dipole on perpendicular bisector of dipole at 'x' distance from centre of dipole



$r^2 = l^2 + x^2$

$r = \sqrt{l^2 + x^2}$

$$\vec{E}_{-q} = \frac{Kq}{(\sqrt{l^2+x^2})^2} = \frac{Kq}{(l^2+x^2)}$$

and,

$$\vec{E}_{+q} = \frac{Kq}{(l^2+x^2)}$$

Thus, magnitude of $\vec{E}_{-q}$ and $\vec{E}_{+q}$ are equal, so, the net electric field $\vec{E}_{net}$ is equal to

$$E_{net} = 2E\cos\theta$$

$\left[\because \cos\theta = \frac{B}{H} = \frac{l}{r}\right.$

$$E_{net} = \frac{2Kq}{x^2+l^2} \cdot \frac{l}{\sqrt{x^2+l^2}}$$

$$E_{net} = \frac{Kq\,2l}{(x^2+l^2)^{3/2}}$$

$$E_{net} = \frac{KP}{(x^2+l^2)^{3/2}}$$

$\because \left[x >>> l \;;\; l^2 \to 0\right]$

$$E_{net} = \frac{Kp}{x^3}$$

$$\boxed{\vec{E}_{\perp} = \frac{-K\vec{p}}{x^3}}$$

$\vec{E}_{net}$ and $\vec{P}$ is antiparallel direction

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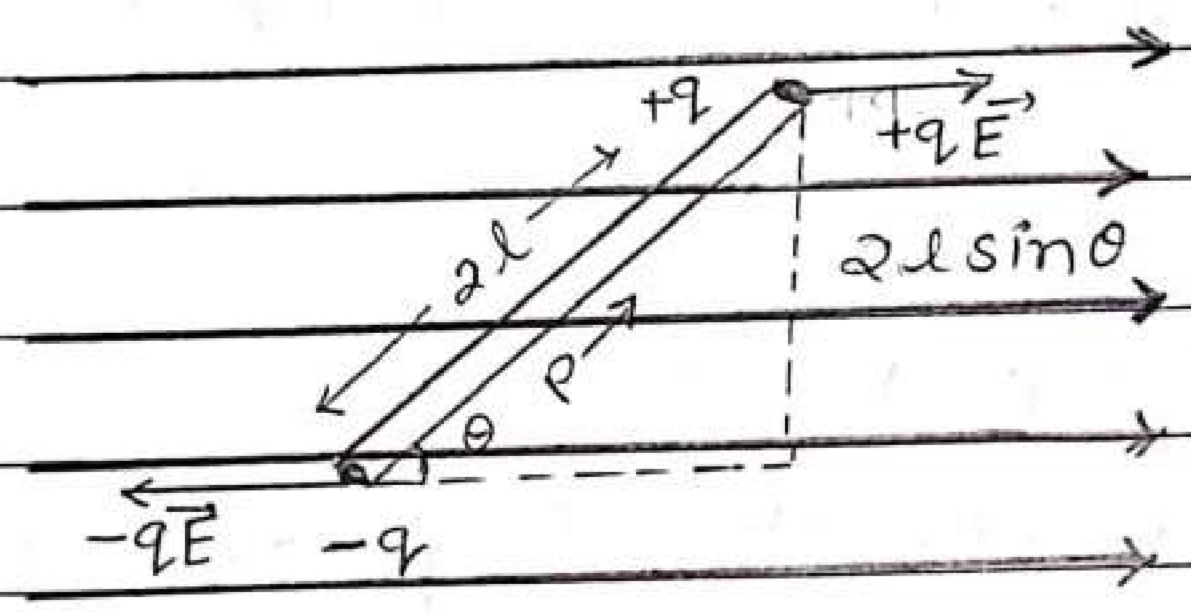
Note → On comparing ;
clearly ,

$$\boxed{E_{axial} = 2\, E_{equatorial}}$$

Hence, at large distance dipole field falls of not as $\frac{1}{r^2}$ but as $\frac{1}{r^3}$.

Torque on a dipole in a uniform $\vec{E}$ field :-



→ Consider an electric dipole consisting of charge +q and -q and of length 2a placed in a uniform electric field $\vec{E}$ making an angle θ with it.
It has dipole moment of magnitude.

$$p = q \times 2a$$

Force exerted on +q by field $\vec{E}$ = $q\vec{E}$ (along $\vec{E}$)
Force " " -q by " $\vec{E}$ = $-q\vec{E}$ (opposite $\vec{E}$)

$$\vec{F}_{total} = 0$$

Net translating force on a dipole in uniform $\vec{E}$ field is zero.
But -q and +q form a couple which exerts a torque.

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~~Torque~~ Torque = force × ⊥ distance between two force.

$\tau = qE \times 2l \sin\theta$

$$\boxed{\tau = pE\sin\theta}$$

As the direction of Torque is perpendicular to both $\vec{p}$ and $\vec{E}$, so,

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

▷ when $\vec{P}$ is along $\vec{E}$, $\theta = 0°$, $\tau = PE\sin\theta = 0$
the dipole is in stable equilibrium.

▷ when $\vec{P}$ is along $\vec{E}$, $\theta = 180°$, $\tau = PE\sin 180° = 0$
The dipole is in unstable equilibrium.

→ The torque will be maximum when dipole is held perpendicular to $\vec{E}$,

$\tau_{max} = PE\sin 90° = PE$

$\tau_{min} = PE\sin 0° = 0$

If $E = 1$ unit, $\theta = 90°$, then $\tau = P$

→ Dipole moment can be defined as the torque acting on an electric dipole, placed perpendicular to a uniform electric field of unit strength.

Dipole in non uniform $\vec{E}$ field :-

In either case, when $\vec{P}$ is parallel to $\vec{E}$ or antiparallel to $\vec{E}$, the net force is zero, but there is a net force on dipole if $\vec{E}$ is not uniform.

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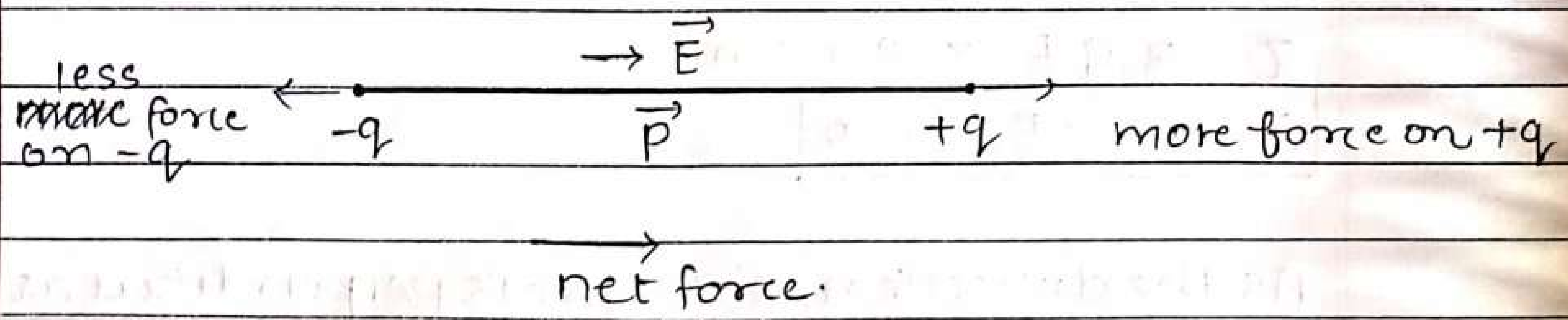
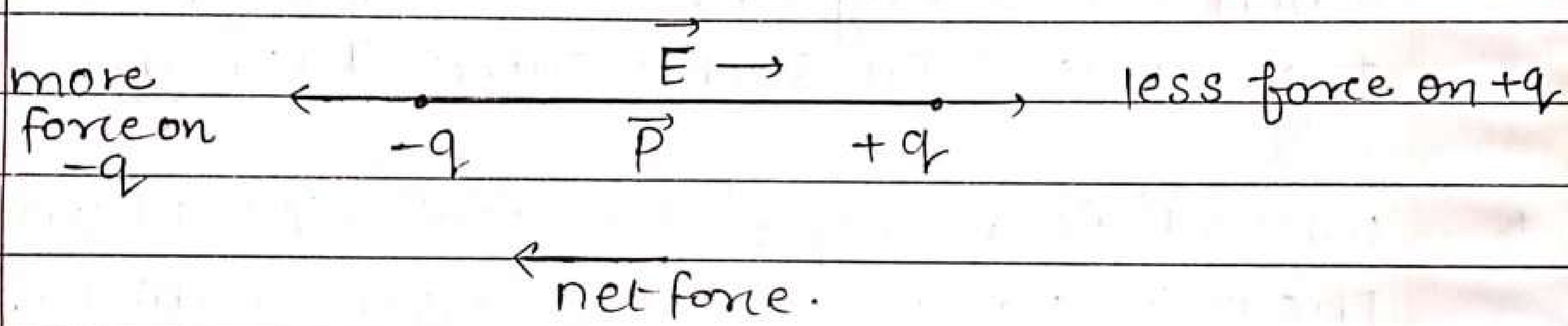
ⓐ when field is increasing along $\vec{P}$:-



Net force on dipole is along $\vec{E}$
Net Torque $= PE\sin\theta = 0$ $[\theta = 0^\circ]$

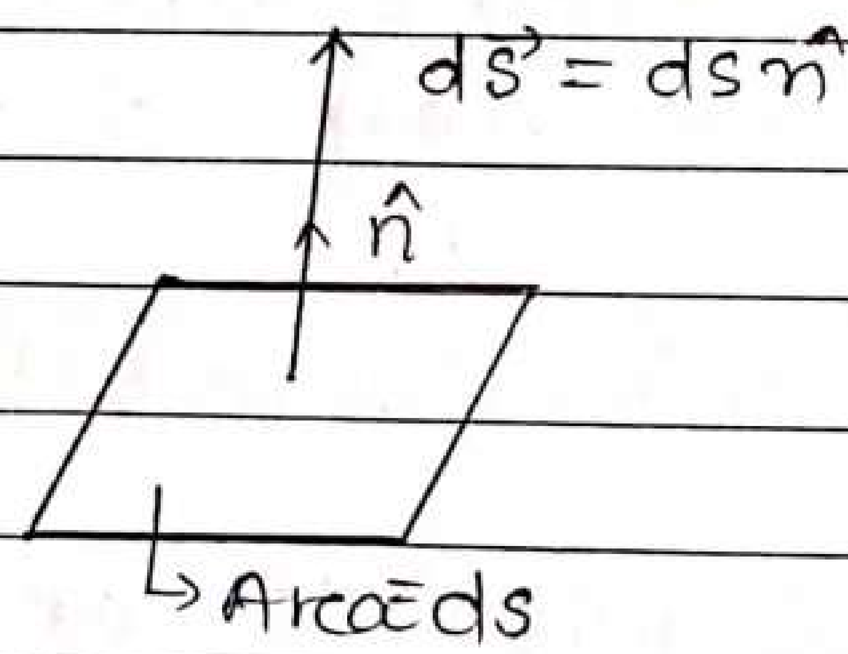
ⓑ when field is decreasing along $\vec{P}$:-



Net force on dipole is opposite to $\vec{E}$.
Net Torque $= PE\sin\theta = 0$ $[\theta = 0^\circ]$

Area Vector :-

Area vector is a vector quantity which is acting outward perpendicular on the surface. The direction of a planar area vector is perpendicular / normal to the plane.



Electric Flux :-

the electric flux through a given area held inside an electric field is the measure of total number of electric lines of force passing normally through that area.

$$\boxed{\phi_E = \vec{E}\cdot\vec{A} \quad \text{or} \quad EA\cos\theta}$$

→ Scalar quantity.
→ SI unit is Nm^2C^{-1} / Vm
→ Dimension is $[ML^3T^{-3}A^{-1}]$

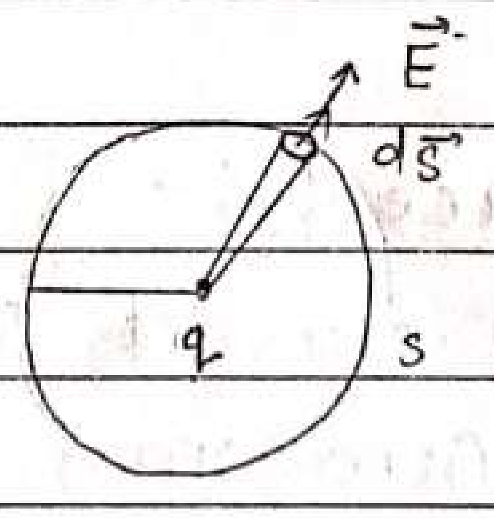
Gauss's Theorem :-

It gives a relationship between total flux passing through any closed surface and net charge enclosed by the closed surface.

Gauss theorem states that the total flux through a closed surface is $1/\varepsilon_0$ times the net charge enclosed by the closed surface.

$$\boxed{\phi_E = \oint_S \vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_0}}$$

Proof :



let a spherical guassian surface S with radius r centered on q.

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Electric Field at any point on S is

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

Also, area element points radially outwards, so it is parallel to $\vec{E}$; $\theta = 0°$

$\because$ flux through area $d\vec{s}$ is

$$d\phi_E = \vec{E} \cdot d\vec{s} = E ds \cos 0° = E ds$$

Total flux through ~~area~~ surface S is

$$\phi_E = \oint_S d\phi_E$$

$$\phi_E = \oint_S E ds$$

$$\phi_E = E \oint_S ds$$

$$\phi_E = E \times \text{Total area of sphere}$$

$$\phi_E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \cdot 4\pi r^2$$

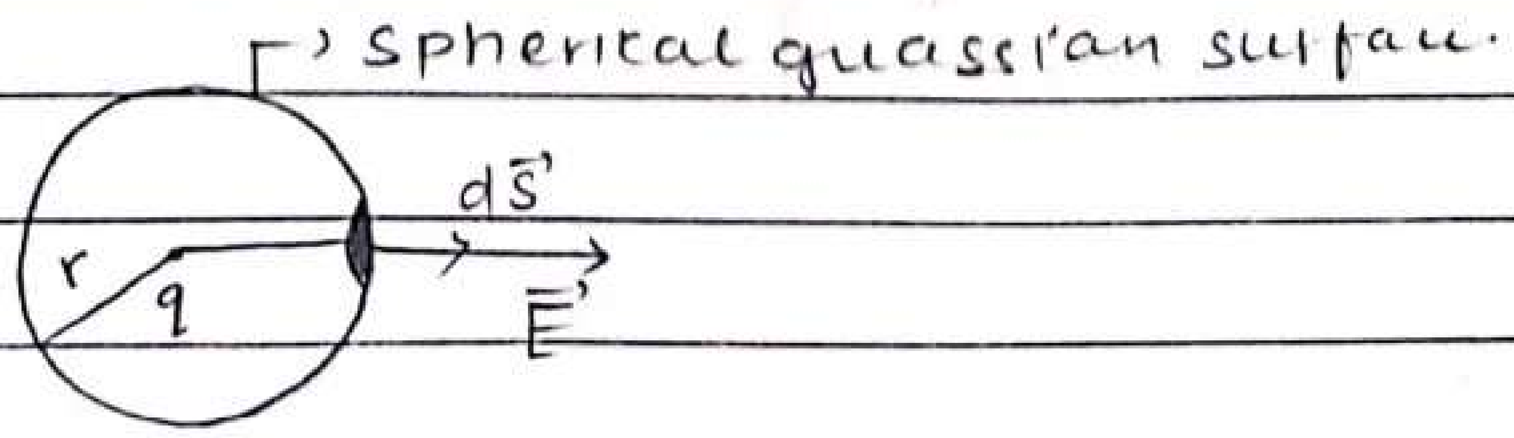
$$\phi_E = \frac{q}{\varepsilon_0}$$

This proves guass theorem.

Guassian Surface :-

Any hypothetical closed surface enclosing a charge is called Guassian Surface.

Coulomb's law from Gauss Theorem :-



$\vec{E}$ and $d\vec{s}$ at any point on S are directed radially outward.

Flux through area S is -

$$\phi_E = \vec{E}\cdot d\vec{s} = E\,ds\cos 0^\circ = E\,ds$$

Net flux through closed surface S is

$$d\phi_E = \oint_S \vec{E}\cdot d\vec{s} = \oint_S E\,ds = E\oint_S ds$$

$$d\phi_E = E \times 4\pi r^2$$

By Gauss theorem,

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

Force on point charge q_0 if placed on surface S will be

$$F = q_0 E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q q_0}{r^2}$$

This proves the Coulomb's law.

Application of Gauss theorem :-

1). Field due to an infinitely long straight uniformly

Consider a thin infinitely long straight thin wire having uniform charge density λ Cm^{-1}.

To determine the field at a distance r from the line charge, we choose a cylindrical Gaussian surface

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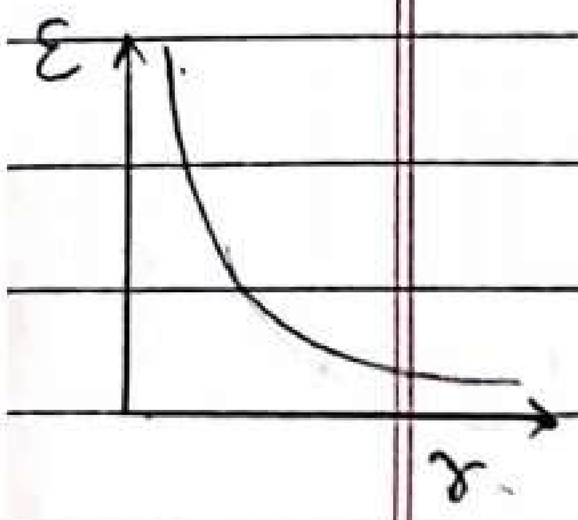
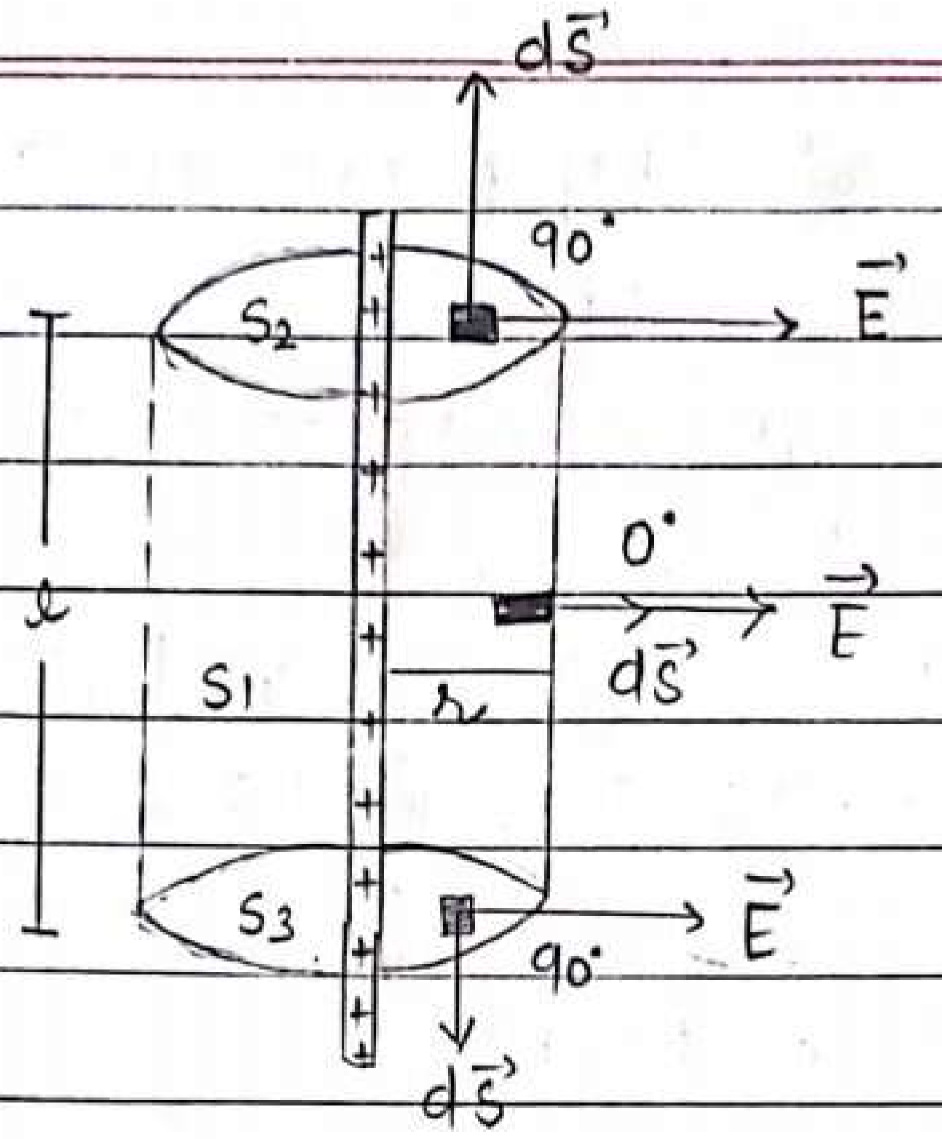
E v/s r graph





As shown in figure it has curved surface S_1 and flat circular ends S_2 and S_3.

$$\phi_E = \oint_S \vec{E}\cdot d\vec{s} = \oint_{S_1} \vec{E}\cdot d\vec{s} + \oint_{S_2} \vec{E}\cdot d\vec{s} + \oint_{S_3} \vec{E} d\vec{s}$$

$$= \oint E\,dS_1 \cos 0^\circ + \oint E\,dS_2 \cos 90^\circ + \int E\,dS_3 \cos 90^\circ$$

$$= E\int dS_1 + 0 + 0$$

$= E \times$ area of curved surface.

$$\phi_E = E \times 2\pi r l \quad \text{— (1)}$$

using Gauss theorem,

$$(2) \text{ — } \phi_E = \frac{q}{\varepsilon_0}, \text{ we get}$$

$$E \cdot 2\pi r l = \frac{q}{\varepsilon_0} \qquad [\because q = \lambda L]$$

$$E \cdot 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

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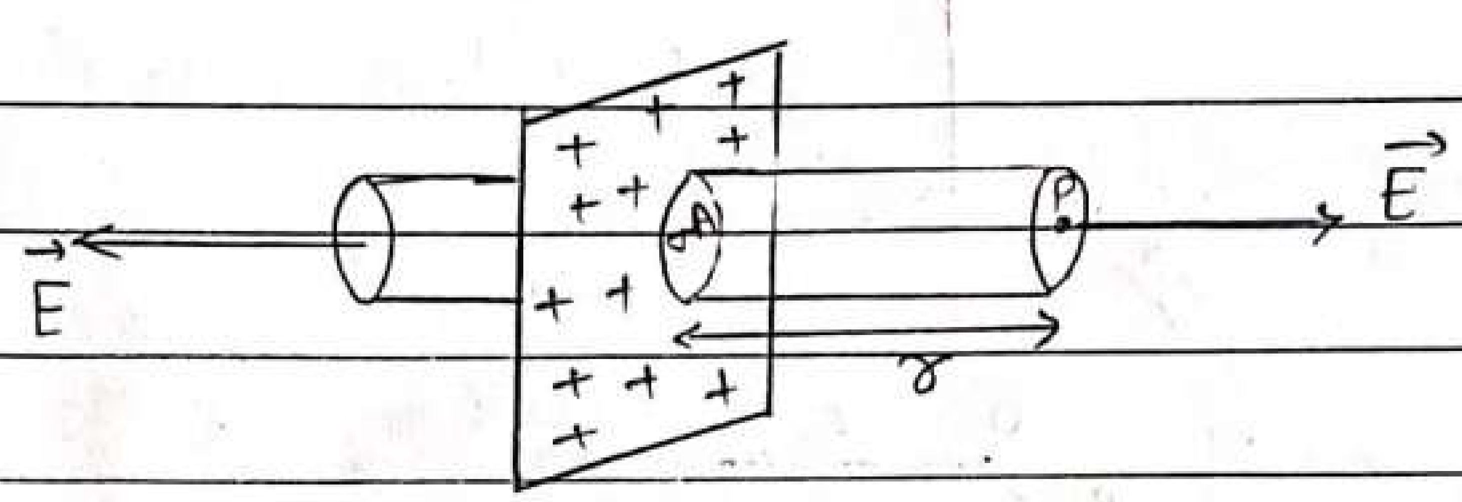
2). Electric field due to a uniformly charged infinite plane sheet.



Consider a thin plane sheet of charge with uniform surface charge density σ. we have to calculate $\vec{E}$ field from r distance to at point P.

we choose cylindrical guassian surface of cross-sectional area A and length 2r with its axis's ⊥ to sheet.

As lines of force are parallel to curved surface of cylinder, the flux through curved surface is zero.

Flux through plane end,

$$\phi_E = EA + EA = 2EA$$

Charge enclosed by a guassian surface, $q = \sigma A$

According to guass theorem,

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$\therefore 2EA = \frac{\sigma A}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

In vector, $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$

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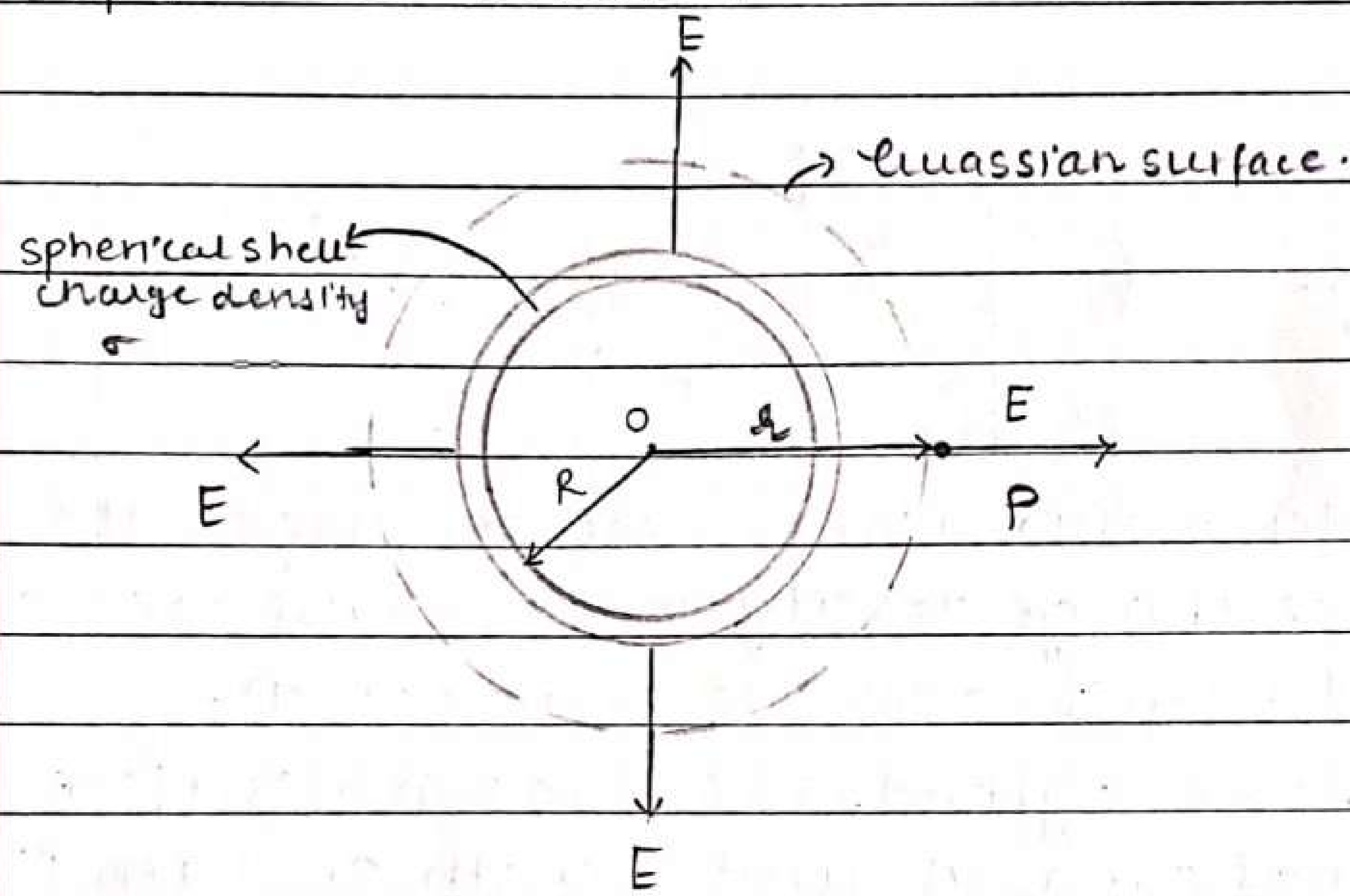
③ Field due to a uniformly charged thin spherical shell :-



ⓐ when point P lies outside the spherical shell.

To determine electric field at any point P at a distance of radius r as Guassian surface.

Total charge q inside guassian surface is charge on the shell of radius R and area $4\pi R^2$.

$$\therefore q = 4\pi R^2 \sigma$$

Flux through guassian surface:

$$\phi_E = \oint \vec{E} \cdot d\vec{S}$$

$$\phi_E = E \times \text{area.} \qquad [\text{For } R < r]$$

$$\phi_E = E \times 4\pi r^2$$

By guass theorem;

$$\phi_E = \frac{q_{inside}}{\varepsilon_0}$$

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}} \Rightarrow \vec{E} = \frac{q}{4\pi r^2} \hat{r}$$

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ⓑ when poin P lies on spherical shell :-

The guassian surface just encloses the charged spherical shell. $[r = R]$

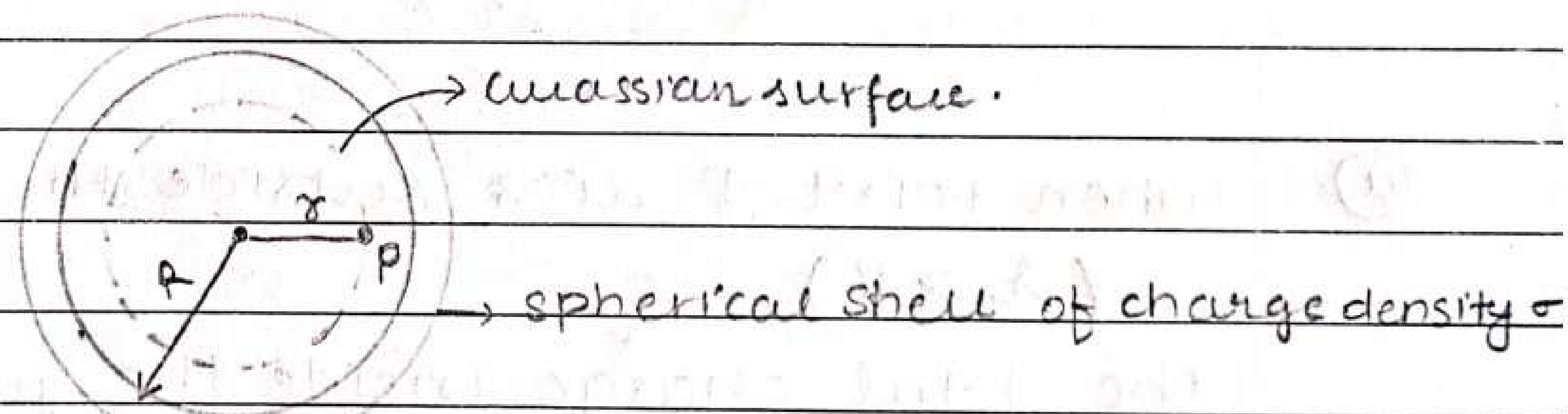
By guass theorem ;

$$E \times 4\pi R^2 = \frac{q}{\varepsilon_0}$$

$$\boxed{E = \frac{q}{4\pi \varepsilon_0 R^2}}$$

$$E = \frac{\sigma}{\varepsilon_0} \quad \because [q = 4\pi R^2 \sigma]$$

ⓒ when point P lies ~~out~~ inside the surface of spherical shell :



we know that q inside guasian surface is zero

$q = 0$.

Flux through guassian surface

$$\phi_E = E \times 4\pi r^2$$

Applying guass theorem,

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$E \times 4\pi r^2 = 0$$

$$\boxed{E = 0}$$

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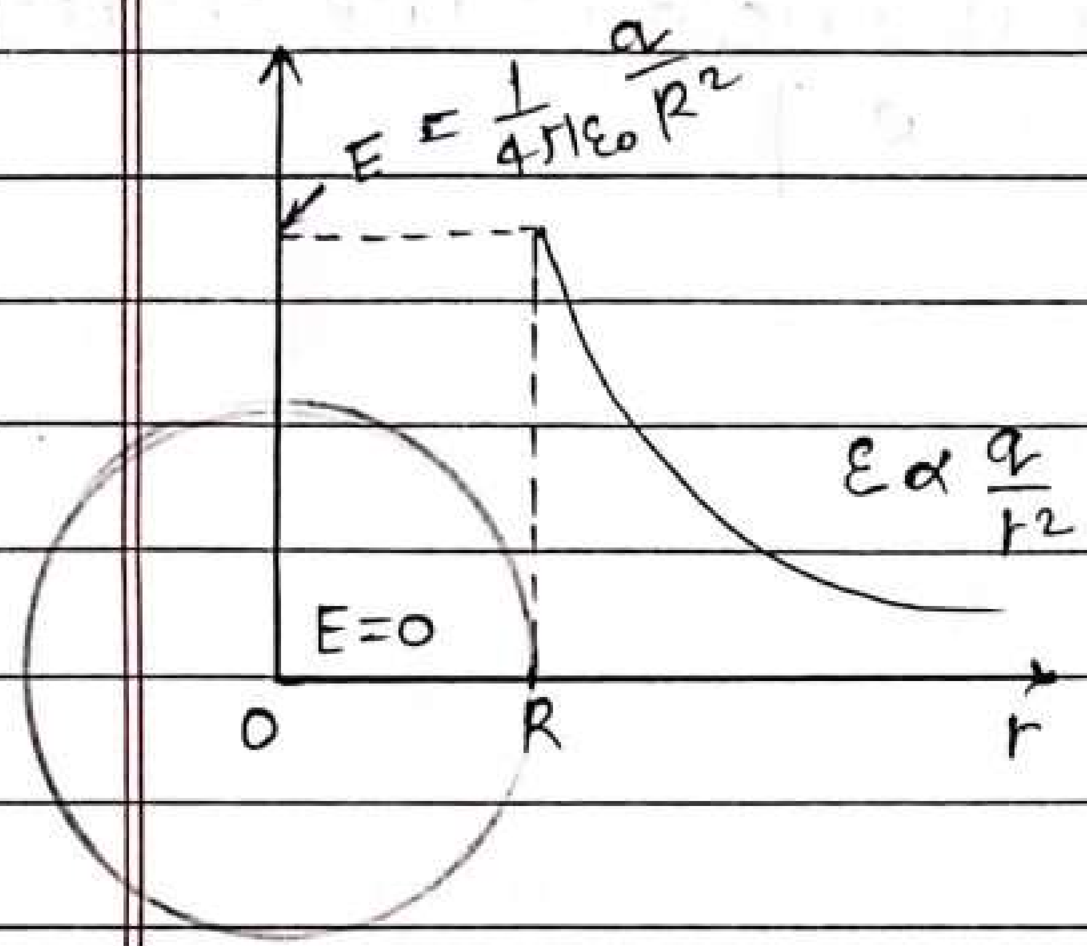
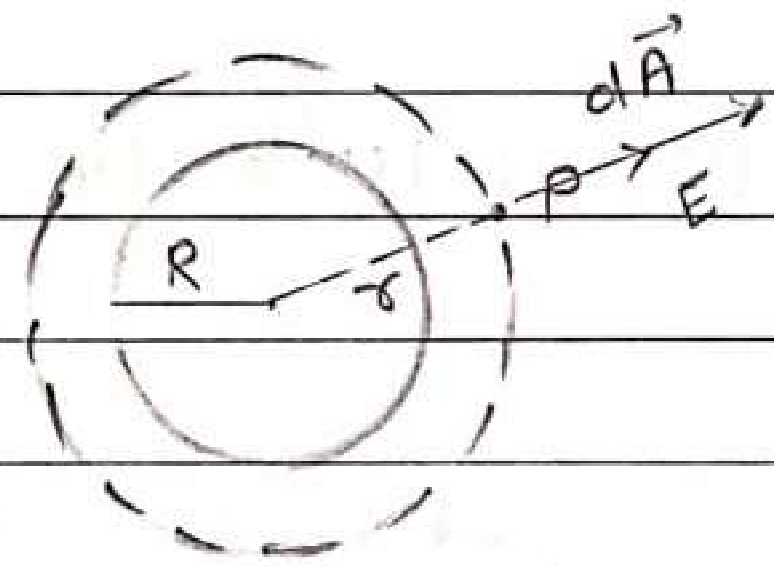
Variation of E with r for a spherical shell of charge.



E varies with distance r from centre of shell of radius R. E is zero from r = 0 to r = R.

④ Field Due to a Uniformly Charged insulating sphere :-



ⓐ when point P lies outside the sphere.
$(r > R)$

The total charge inside the guassian surface is the charge inside the ~~surface~~ ~~sphere~~ the guassian surface is the charge inside the sphere of radius R.

$$\rho = \frac{q}{V}$$

$$q = \frac{4\pi R^3 \rho}{3}$$

Flux through the guassian surface,

$$\phi_E = E \times 4\pi r^2$$

By guass theorem,

$$\phi_E = \frac{q_{inside}}{\varepsilon_0}$$

$$\phi_E = \frac{4/3 \pi R^3 \rho}{\varepsilon_0} \quad \text{—②}$$

$$\because \quad E \times 4\pi r^2 = \frac{4}{3}\frac{\pi R^3 \rho}{\varepsilon_0}$$

$$E \times r^2 = \frac{R^3 \rho}{3 \times \varepsilon_0}$$

$$\boxed{E = \frac{\rho R^3}{3 \varepsilon_0 r^2}}$$

or

$$\boxed{E = \frac{q}{4\pi \varepsilon_0 r^2} \hat{r}}$$

(b) When point P lies on the sphere.

The guassian surface just encloses the charged sphere. Applying the guass theorem,

$$\phi_E = E \times 4\pi R^2 \quad \text{—①}$$

also, $\phi_E = \frac{q_{inside}}{\varepsilon_0}$ —②

$$E \times 4\pi R^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{4\pi R^2 \varepsilon_0} \quad [\text{for } r = R]$$

$$\boxed{E = \frac{q}{4\pi \varepsilon_0 R^2}}$$

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(c) when Point P lies inside sphere.

The charge enclose by guassian surface of radius r is -

$\rho = \frac{q}{V}$; $q = \frac{4}{3}\pi r^3 \times \rho$

flux through guassian surface ;

$\phi_E = E \times 4\pi r^2$ —— (1)

By Guass ~~Law sur~~ theorem ;

$\phi_E = \frac{q}{\varepsilon_0} = \frac{4/3 \pi r^3 \rho}{\varepsilon_0}$

$\phi_E = \frac{4\pi r^3 \rho}{3\varepsilon_0}$ —— (2)

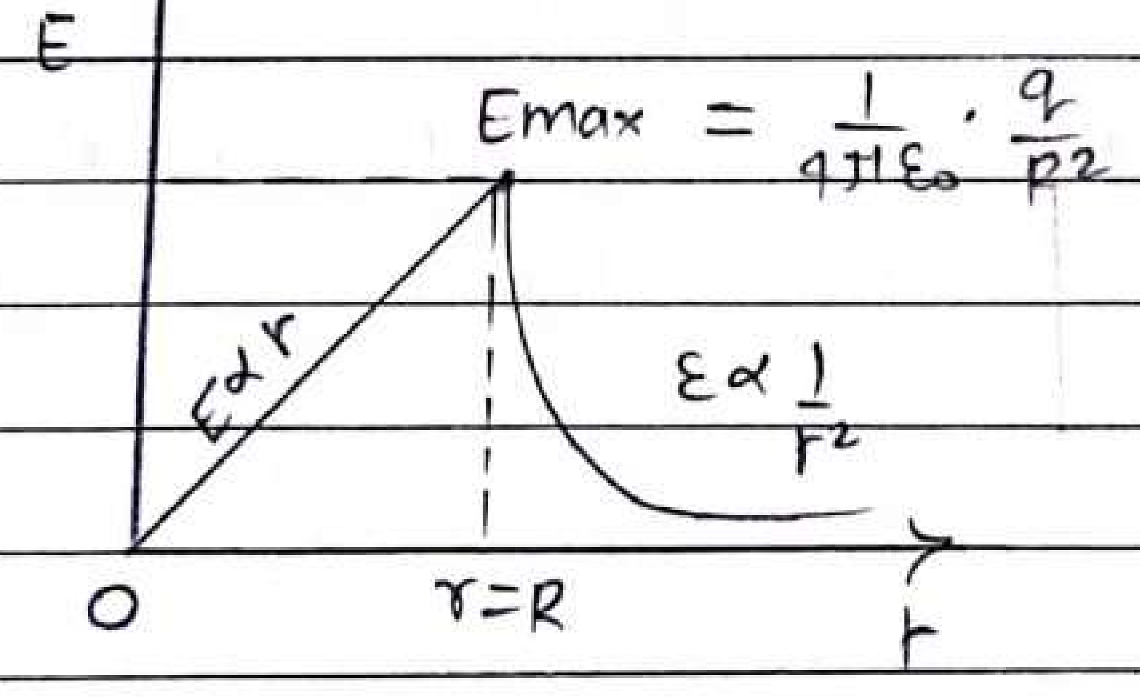
from (1) and (2) -

$E \times 4\pi r^2 = \frac{4\pi r^3 \rho}{3\varepsilon_0}$

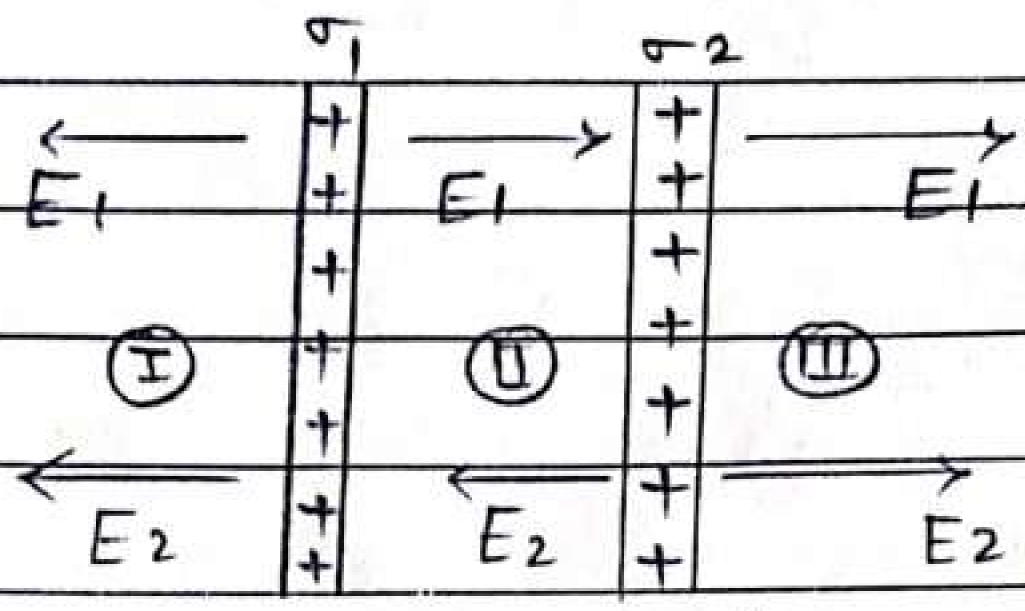
$E = \frac{4 \rho r}{3\varepsilon_0}$

$$\boxed{E = \frac{\rho r}{3\varepsilon_0}}$$

Graph !



Electric Field of two positively charged parallel plates.



In region I:

$$\vec{E_1} = \frac{-\sigma_1}{2\varepsilon_0}\hat{r} \quad , \quad E_2 = -\frac{\sigma_2}{2\varepsilon_0}\hat{r}$$

from superposition principal:

$$\vec{E_I} = \vec{E_1} + \vec{E_2}$$

$$\vec{E_I} = \frac{-\hat{r}}{2\varepsilon_0}(\sigma_1 + \sigma_2)$$

In region II:

$$\vec{E_1} = \frac{\sigma_1}{2\varepsilon_0}\hat{n} \, , \quad \vec{E_2} = -\frac{\sigma_2}{2\varepsilon_0}\hat{r}$$

$$\vec{E_{II}} = \frac{\hat{r}}{2\varepsilon_0}(\sigma_1 - \sigma_2)$$

In region III:

$$\vec{E_1} = \frac{\sigma_1}{2\varepsilon_0}\hat{r} \quad , \quad \vec{\mathcal{E}_2} = \frac{\sigma}{2\varepsilon_0}\hat{r}$$

$$\vec{E_{III}} = \frac{\hat{r}}{2\varepsilon_0}(\sigma_1 + \sigma_2)$$