Set Theory

1. Fundamentals & Representation

Definitions

- **Set**: A well-defined collection of distinct objects.
- **Representation**:

1. Roster/Tabular Form: Elements listed in braces, e.g., $A = \{1, 2, 3\}$.

2. Set-Builder Form: defined by a property $P(x)$, e.g. $A = \{x : x \in N, x < 4\}$.

Standard Notations (Important)

- N: Natural numbers $\{1, 2, 3, \ldots\}$
- W: Whole numbers $\{0, 1, 2, \ldots\}$
- Z or I: Integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$

- Q: Rational numbers $\{p/q : p, q \in Z, q \neq 0\}$
- T or I^c: Irrational numbers

- R: Real numbers (Rational + Irrational)
- C: Complex numbers

Types of Sets

- Empty/ Null / Void Set (ϕ or $\{\}$): Contains no elements.

- Note: ϕ is a subset of every set.

- Singleton Set: Contains exactly one element.
- Finite Set: Countable number of elements. cardinal number $n(A)$ = finite.

- Infinite Set: Uncountable number of elements.

- Equal Sets ($A = B$): Every element of A is in B and every element of B is in A.

- Equivalent Sets: $n(A) = n(B)$ (same number of elements, elements need not be identical).

2. Subsets & Power Sets

Subsets

- $A \subseteq B$: Every element of A is also in B.
- Proper Subset ($A \subset B$): $A \subseteq B$ but $A \neq B$.

- Total Number of Subsets: If $n(A) = m$, total subsets $= 2^m$.

- Total Proper Subsets: $2^m - 1$ (excluding A itself).

Power Set P(A)

- The set of all subsets of A.
- If $n(A) = m$, then $n(P(A)) = 2^m$.

Example : If $A = \{1, 2\}$, $P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$.

Intervals (subsets of R)

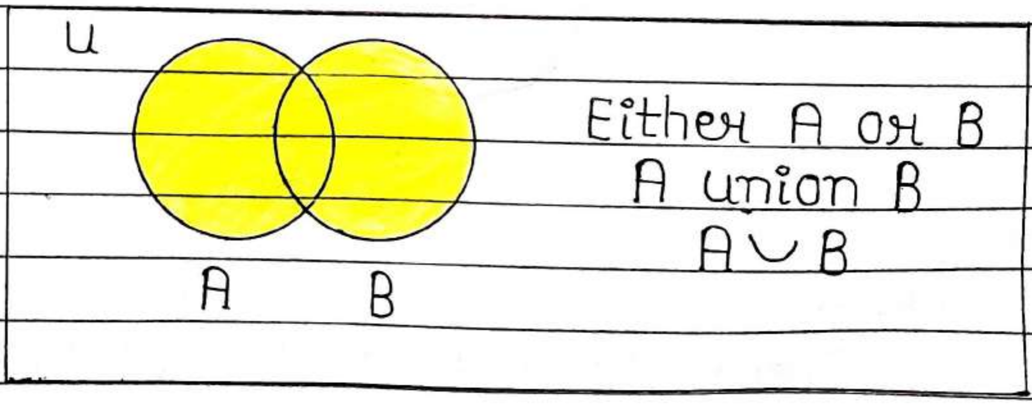
- Open : $(a,b) = \{x : a < x < b\}$
- closed : $[a,b] = \{x : a \le x \le b\}$
- Semi-Open : $[a,b) = \{x : a \le x < b\}$

3. Operations on Sets & Venn Diagrams

Union (A ∪ B)

Elements in A OR B (or both).

$$A \cup B = \{x : x \in A \vee x \in B\}$$

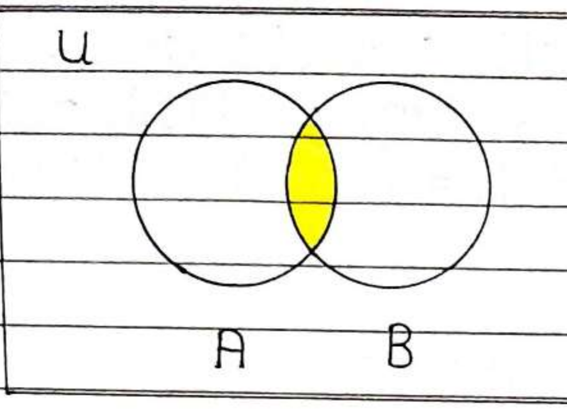


Intersection (A ∩ B)

Elements common to both A AND B.

$$A \cap B = \{x : x \in A \wedge x \in B\}$$

- Disjoint Sets : If $A \cap B \neq \phi$.

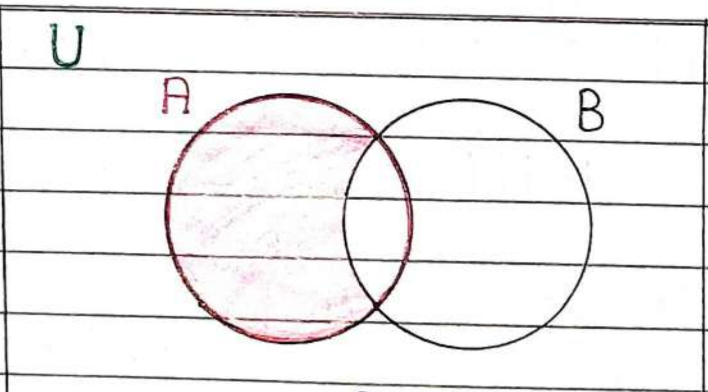


Both A & B
A intersects B
$A \cap B$

Difference $(A-B)$
Elements in A but NOT in B.

$$A-B = \{x : x \in A \wedge x \notin B\}$$

- Important: $A-B = A \cap B'$

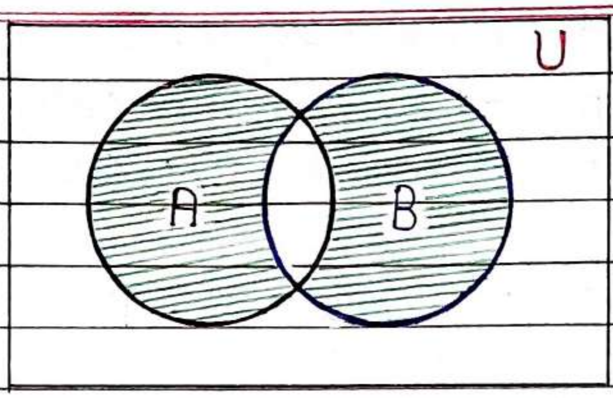


$A-B$

Symmetric Difference $(A \Delta B)$
Elements in A or B, but NOT in both.

$$A \Delta B = (A-B) \cup (B-A)$$

$$A \Delta B = (A \cup B) - (A \cap B)$$



Complement (A' or A^c)

Elements in Universal Set U but NOT in A.

$$A' = U - A = \{x : x \in U \wedge x \notin A\}$$

4. Algebra of Sets (Laws)

These are critical for simplifying set expressions in JEE problems.

1. Idempotent Laws :
   - $A \cup A = A$
   - $A \cap A = A$

2. Identity Laws :
   - $A \cup \phi = A$
   - $A \cap U = A$

3. Commutative Laws :
   - $A \cup B = B \cup A$
   - $A \cap B = B \cap A$

4. Associative Laws :
   - $(A \cup B) \cup C = A \cup (B \cup C)$
   - $(A \cap B) \cap C = A \cap (B \cap C)$

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5. Distributive Law (Very Important):
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

6. De Morgan's Laws (Crucial):
- $(A \cup B)' = A' \cap B'$
- $(A \cap B)' = A' \cup B'$

7. Absorption Laws:
- $A \cup (A \cap B) = A$
- $A \cap (A \cup B) = A$

Advanced Properties of Symmetric Difference (Δ).

- $A \Delta A = \phi$
- $A \Delta \phi = A$
- $A \Delta B = B \Delta A$ (commutative)
- $(A \Delta B) \Delta C = A \Delta (B \Delta C)$ (Associative)
- $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$ (Distributive)
- If $A \Delta B = A \Delta C$, then $B = C$ (cancellation Law)

5. Cardinality Formulas

For Two Sets (A, B)

1. Basic Formula:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

2. If Disjoint:

$$n(A \cup B) = n(A) + n(B)$$

3. Difference :

$$n(A-B) = n(A) - n(A \cap B) = n(A \cup B) - n(B)$$

For Three Sets (A, B, C)

1. Union of 3 Sets :

$$n(A \cup B \cup C) = \sum n(A) - \sum n(A \cap B) + n(A \cap B \cap C)$$

(Sum of singles - Sum of doubles + Triple)

JEE Special Cases (Regions)

Let $n(P)$ be elements in exactly Set P.

1. Exactly ONE of the Sets :

$$= \sum n(A) - 2\sum n(A \cap B) + 3n(A \cap B \cap C)$$

2. Exactly TWO of the Sets :

$$= \sum n(A \cap B) - 3n(A \cap B \cap C)$$

3. None of the Sets :

$$n(A' \cap B' \cap C') = n(U) - n(A \cup B \cup C)$$

6. Optimization (Min/Max Values) - JEE Critical

For problems asking "At least how many ..." or "At most how many ..."

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For Union $n(A \cup B)$

- Maximum : $n(A) + n(B)$ (when $A \cap B = \phi$)
- Minimum : $\max(n(A), n(B))$ (when one set is subset of other)

For Intersection $n(A \cap B)$

- Maximum : $\min(n(A), n(B))$ (when one set is subset of other)

- Minimum : $n(A) + n(B) - n(U)$ (Derived from $n(A \cup B) \leq n(U)$)

7. Important Relations & Logic Symbols

- $A \subseteq B \Leftrightarrow A \cap B = A \Leftrightarrow A \cup B = B \Leftrightarrow A - B = \phi$.
- $n(A \times B) = n(A) \times n(B)$.

Symbol	Meaning
$\forall$	For all / For every
$\exists$	There exists
$\in$	Belongs to
$\subset$	Proper Subset
$\Longrightarrow$	Implies
$\Longleftrightarrow$	If and only if (iff)
$\ni$	Such that
$\subseteq$	Subset