

Electric Charges & fields

Electrostatics is study of electric charge at rest.

Electric charge :-

Electric charge is a fundamental property of elementary particles that causes them to exert electric forces on each other.

It is scalar quantity. It is denoted by Q or q . Its magnitude is equal to product of current and time.

$$\text{i.e.; } Q = IT$$

$$\begin{aligned} \text{SI unit} &= \text{ampere} \times \text{sec} \\ &= \text{As} \\ &= \text{coulomb} \\ &= \text{C} \end{aligned}$$

$$\text{Dimension} = [AT] \text{ or } [IT]$$

Other unit of charge

1. CGS Unit = stat coulomb
= esu
= electro static unit

Where,

$$\begin{aligned} 1\text{C} &= 3 \times 10^9 \text{ stat coulomb} \\ 1\text{C} &= 3 \times 10^9 \text{ esu} \end{aligned}$$

$$1 \text{ esu} = 1 \text{ stat coulomb}$$

$$1 \text{ esu} = \frac{1}{3 \times 10^9} \text{ C.}$$

$$1 \text{ st-C} = \frac{1}{3 \times 10^9} \text{ C.}$$

Types of charge :-

(i) Positive charge - If a body loses electron then charge on the body is called positive charge.

So, positive charge = loss of electron
= decrease in electron
= Small decrease in mass.

(ii) Negative charge - If a body gains electron then charge of the body is called negative charge.

So, Negative charge = Gain of electron
= Increase of electron
= Small increase in mass.

Examples

(i) When comb is rubbed over then dry hair loses electron and the comb gains electron.

So, dry hair becomes positively charged and the comb becomes negatively charged.

(ii) Glass rod $\approx$ Woolen cloth



→ Fundamental Law of Electrostatics :-

According to this law, like charge repel and unlike charge attract each other.

$$q_e = -1.6 \times 10^{-19} \text{ C}$$

$$q_p = +1.6 \times 10^{-19} \text{ C}$$

$$q_n = 0$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

$$m_n = 1.67 \times 10^{-27} \text{ kg}$$

also, charge is always associated with mass.
i.e., charge can't exist without mass.

* Classification of material with respect to charge :-

(i) Conductor :- Conductors are materials that allow electric charge to flow easily through them. They have a large number of free electrons, which facilitate the easy movement of charge, making them good conductors of electricity.

eg :- Metals, human and animal bodies etc.

(ii) Insulator :- Insulators are materials that resist the flow of electric charge because they lack free electrons. As a result, they offer very high electrical resistance and do not conduct electricity.

eg :- Non metals like glass, diamond, plastic.

(iii) Semi-conductor :- It behaves like an insulator

at low temperatures, but as the temperature increases, it starts conducting electricity and act like a conductor.

eg:- Silicon, Germanium etc.

It most object are electrically neutral, which means that they have an equal number of positive and negative charges. In order to charge an object, one has to alter the charge balance of positive & negative charges.

There are three ways to do it —

(i) Charging by friction :- The method of charging in which when two bodies are rubbed to each other then one body becomes positively charged and another becomes negatively charge is called charging of friction.

In this one body looses electrons and another gains electron.

(ii) Charging by Induction :- The process of charging in which a body can be charge without touching. it is called charging by induction.

(iii) charging by conduction :- The charging by conduction process involves touching of a charged particle to a conductor material (say rod). It attracts the negative charge and leaving the other sphere positively charged.

Now the spheres are separated before the rod is removed and thus separating positive and negative charge. The charge on the sphere will be rearranged and uniformly distributed over.

This method is useful for charging conductors.

* Difference between charge and mass :-

Charge	Mass
1. It can be positive or negative, zero.	1. It is always positive.
2. It is always conserved.	2. It is not conserved by itself. Some mass changes into energy.
3. Charge is always associated with mass.	3. A body possessing mass may not have any net charge.
4. Charge does not depend on speed.	4. It depends on speed. In fact mass increases with speed.

Charge

Mass

5. Charge is always quantised.

5. Quantisation of mass not yet established.

6. Electrostatic force between two charges may be attractive or repulsive.

6. Interaction between two masses always attractive.

* Basic properties of electric charge :-

Charge has three basic properties -

- Additivity
- Quantisation
- Conservation

- Additivity of charge means that the total charge of a system is algebraic sum of all individual charges located at different points inside a system.
If a system contains charge $q_1, q_2, \dots, q_n$ then total charge is -

$$Q = q_1 + q_2 + \dots + q_n$$

- Quantisation of electric charge means that the total charge (Q) of a body is always an integral multiple of basic quantum of charge.

$$Q = \pm n \cdot e \quad \text{where ;}$$

Q = total charge of a body

n = integer value

e = charge on one electron
($1.6 \times 10^{-19} \text{ C}$).

• Conservation of charge :

The law of conservation states that —

- (i) The total charge of an isolated system remain conserved.
- (ii) The electric charge can neither be created nor destroyed, they can be only transferred from one body to another.

Example of conservation of charge :

- ▶ When a glass rod is rubbed with a silk cloth it develops a positive charge. But at the same time, silk cloth develops equal negative charge. Thus, net charge before and after is zero.

* Coulomb's Law :

The electrostatic force acting between two charges is directly proportional to product of charge and inversely proportional to square of distance between them,

i.e;

$$q_1 \quad \quad \quad q_2$$

$$K \text{ --- } r \text{ --- } K$$

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\Rightarrow F = \frac{k \cdot q_1 q_2}{r^2}$$

Here, k = Electrostatic constant
 = Coulomb's constant
 = $9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$

r = distance

Again,

$$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

$$\therefore F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

→ One coulomb is the amount of charge that repels equal and similar charge with a force of $9 \times 10^9 \text{ N}$ when placed in vacuum at 1m distance between them.

k depends on nature of medium between two charges.

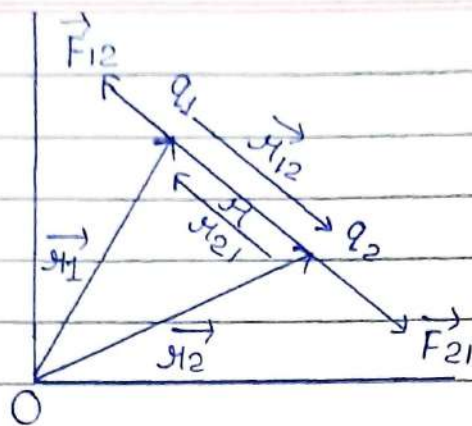
Dimension of k : $[ML^3T^{-4}A^{-2}]$

Unit of k : Nm^2C^{-2}

Dimension of permittivity (ϵ_0) : $[M^{-1}L^{-3}T^4A^2]$

Unit of ϵ_0 = $\text{N}^{-1}\text{m}^{-2}\text{C}^2$.

Coulomb's Law in vector form :-



Let the position vector of charges q_1 and q_2 be $\vec{r}_1$ and $\vec{r}_2$ respectively.

We denote the force on q_1 due to q_2 by $\vec{F}_{12}$ and the force on q_2 due to q_1 by $\vec{F}_{21}$.

$$\vec{F}_{21} = -\vec{F}_{12}$$

Here, $\vec{a}_1 + \vec{a}_{12} = \vec{a}_2$ $\vec{a}_2 + \vec{a}_{21} = \vec{a}_1$

$\vec{a}_{12} = \vec{a}_2 - \vec{a}_1$ also, $\vec{a}_{21} = \vec{a}_1 - \vec{a}_2$

and $\vec{a}_{12} = -\vec{a}_{21}$

The magnitude of vector $\vec{a}_{12}$ and $\vec{a}_{21}$ is denoted by $|\vec{a}_{12}|$ and $|\vec{a}_{21}|$ and their unit vector is given as -

$$\hat{a}_{12} = \frac{\vec{a}_{12}}{|\vec{a}_{12}|} \quad \text{and} \quad \hat{a}_{21} = \frac{\vec{a}_{21}}{|\vec{a}_{21}|}$$

So, Force $\vec{F}_{12}$ can be given as;

$$\vec{F}_{12} = \left(\frac{k q_1 q_2}{r_{12}^2} \right) \hat{a}_{12}$$

$$\vec{F}_{12} = \left(\frac{k q_1 q_2}{r_{12}^2} \right) \frac{\vec{a}_{12}}{|\vec{a}_{12}|}$$

$$\vec{F}_{12} = \left(\frac{k q_1 q_2}{r_{12}^3} \right) \vec{a}_{12}$$

Similarly, $\vec{F}_{21}$ can be given as;

$$\vec{F}_{21} = \left(\frac{k q_1 q_2}{|r_{12}|^3} \right) \vec{r}_{12}$$

- $q_1 q_2 > 0 \rightarrow$ repulsion
- $q_1 q_2 < 0 \rightarrow$ attraction

Importance of vector form -

1. As $\vec{r}_{21} = -\vec{r}_{12}$, therefore $\vec{F}_{12} = -\vec{F}_{21}$ two charge exert equal and opposite force. So, Coulombian force obey Newton's 3rd law of motion.
2. Coulombian force act line joining two charges, so they are central forces.

Limitation of Coulomb's law :-

- 1.) The electric charge must be at rest.
- 2.) The electric charge must be point charges i.e; the extension of charges must be much smaller than separation between the charges.
- 3.) The separation between charge must be greater than the nuclear size ($10^{-15}m$) because strong nuclear force dominates electrostatic force.

Dielectric Constant (K) :-

Force in vacuum $\Rightarrow F_e = \frac{k q_1 q_2}{r^2}$

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \quad \text{--- (1)}$$

Force in medium $\Rightarrow F_m = \frac{k q_1 q_2}{r^2}$

$$F_m = \frac{1}{4\pi\epsilon} \cdot \frac{q_1 q_2}{r^2} \quad \text{--- (2)}$$

Dividing (1) by (2), we get -

$$\frac{F_o}{F_m} = \frac{\frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}}{\frac{1}{4\pi\epsilon} \cdot \frac{q_1 q_2}{r^2}} = \frac{\epsilon}{\epsilon_0}$$

Ratio of (ϵ/ϵ_0) is called relative permittivity (ϵ_r) or dielectric constant (K) of given medium.

$$\epsilon_0 = 8.85 \times 10^{-12}$$

$\epsilon_r \text{ or } K = \frac{\epsilon}{\epsilon_0} = \frac{F_o}{F_m}$
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The dielectric constant or relative permittivity of a medium may be defined as the ratio of force between two charged species placed at some distance in free space to the force between same two charges when they are placed distance in a given medium.

$$F_m = \frac{F^0}{K}$$

ϵ (permissivity)
↓

$$F_m = \frac{1}{4\pi\epsilon_0} \cdot \frac{1}{K} \times \frac{q_1 q_2}{r^2}$$

Property of medium which determines the electric force between two charges placed in that medium.

$$F_m = \frac{1}{4\pi\epsilon_0 K} \cdot \frac{q_1 q_2}{r^2}$$

$$K (\text{air}) = 1.00054$$

$$F_0 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$K (\text{water}) = 80$$

Similarity And Dissimilarity between F_E and F_G :-

→ Similarity

- 1) Both forces obeys inverse square law.
- 2) Both proportional to product of charges and masses.
- 3) Both act along line joining centers (central force).
- 4) Conservation force.
- 5) Both forces can operate in vacuum.

→ Dissimilarity

- 1) F_E is attractive as well as repulsive but F_G is attractive only.
- 2) F_E depends on medium while F_G do not.
- 3) F_E are stronger than F_G .

Principle of super-position :-

According to this principle, when the

number of charge are interacting, the total force on a given charge is the vector sum of forces exerted on it due to all other charges. The force between two charge is not affected by the presence of other charges.

Consider N point charges $q_1, q_2, q_3, \dots, q_n$ placed in vacuum at point whose position vector w.r.t. origin O are $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_n$.

By superposition principle: $F_1 = \vec{F}_{12} + \vec{F}_{13} + \dots + \vec{F}_{1n}$

According to coulomb's law;

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

So, total force on charge q_1 is -

$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} + \frac{q_1 q_3}{r_{13}^2} \hat{r}_{13} + \dots + \frac{q_1 q_n}{r_{1n}^2} \hat{r}_{1n} \right]$$

$$\boxed{\vec{F}_1 = \frac{q_1}{4\pi\epsilon_0} \sum_{i=2}^N \frac{q_i}{r_{1i}^2} \hat{r}_{1i}}$$

In vector,

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)$$

So,

$$\boxed{\vec{F}_1 = \frac{q_1}{4\pi\epsilon_0} \cdot \sum_{i=2}^N \frac{q_i}{|\vec{r}_1 - \vec{r}_i|^3} (\vec{r}_1 - \vec{r}_i)}$$

Formula used for super position —
 $\vec{E}_1 = \vec{E}_2 + \vec{E}_3 + \dots + \vec{E}_n$

$$E = \sqrt{E_1^2 + E_2^2 + 2 E_1 E_2 \cos \theta}$$

* Electric field :-

The region or space around a charge in which another charge experience electrostatic force of attraction or repulsion is known as electric field.

The extension of electric field of a charge is upto infinity. (∞).

Point charge : It is an electric charge considered to exist at a single point, and thus having neither area nor volume.
 or

If the size of charged bodies are very small as compared to distance between them, then we treat it as point charge.

Test charge : A test charge is a charge with a so small magnitude that placing it at a point has negligible effect on field around. It does not create any field of its own.

* Electric field intensity :-

It is defined as the

electrostatic force per unit test charge acting on vanishingly small positive test charge placed at given point.

$$\vec{E} = \lim_{q_0 \rightarrow 0} \frac{\vec{F}}{q_0}$$

$\vec{E} = \frac{\vec{F}}{q_0}$

→ vector quantity

→ its direction is in the direction of electrostatic force acting on positive charge.

→ SI unit is NC^{-1} or Vm^{-1}

→ Dimension is $[MLT^{-3}A^{-1}]$

► Electric field due to a point charge q at a distance r from it is given by

$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$
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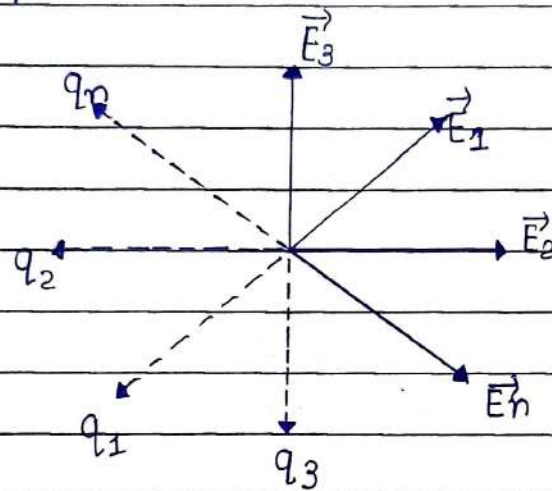
Electric field intensity depends on

- 1) Magnitude of point charge.
- 2) Distance of point from charge.
- 3) Permittivity of medium.

→ If q is positive, E points radially outwards and if q is negative E points, radially inwards.

Electric field due to a system of point charges :-

Consider a system of n charges $q_1, q_2, q_3, \dots, q_n$ due to which field is determined at a point P .



If $r_1, r_2, \dots, r_n$ is the distance of $q_1, q_2, \dots, q_n$ charges from the point P and $E_1, E_2, \dots, E_n$ is electric field then,

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= \left[\frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i^2} \hat{r}_{ip} \right]$$

$$\phi = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{|\vec{r} - \vec{r}_i|}$$

The principle states that the electric field at any point due to a group of point charge is equal to the vector sum of electric fields produced by each charge individually at that point, when all other charge assumed to be absent.

Continuous charge distribution :

Whenever the charge is given to any conductor the charge spread all over the body uniformly.

Volume charge density, $\rho = \frac{dq}{dv} \text{ cm}^{-3}$

Surface charge density, $\sigma = \frac{dq}{ds} \text{ cm}^{-2}$

Linear charge density, $\lambda = \frac{dq}{dl} \text{ cm}^{-1}$

Variation of E with q and μ

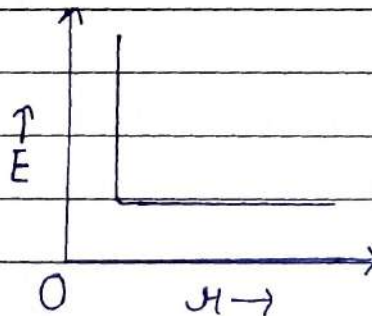
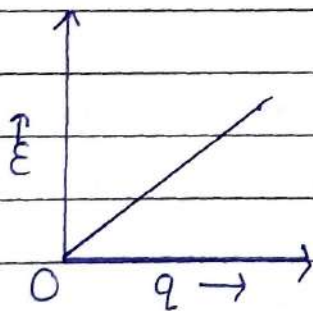
$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{\mu^2}$$

also,

$$E \propto q$$

$$E \propto \frac{1}{\mu^2}$$

Graph :



Electric field lines are the imaginary line in a region of space time along with a free (q_0) positive charge would move if allowed to do so in an electric field and tangent to which at any point gives direction of electric field at that point.

Properties of Electric field lines of force :-

- 1) Lines of force are continuous curves without any breaks.
- 2) No two lines of force can cross each other.
- 3) They start at positive charges and ends at negative charge - they cannot form closed loops.
- 4) The relative closeness of the lines of force indicates the strength of electric field at different points.
- 5) They are always normal to the surface of the conductor.
- 6) They have a tendency to contract length wise and expand laterally.

Force exerted on charge q_0 due to continuous charge distribution,

$$\vec{F} = q_0 \int \frac{dq}{r^2} \hat{r}$$

$\vec{E}$ due to continuous charge.

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$

Electric Dipole $\div$ $(+q \text{ and } -q)$

A pair of equal and opposite charges separated by a small distance $(2a)$ is called electric dipole.

Electric Dipole movement $\div$

It measures the strength of electric dipole. The dipole moment of an electric dipole is vector whose magnitude is either charge times the separation between two opposite charge and direction is along the dipole axis from negative to positive charge.

$$\boxed{\vec{P} = q \times 2\vec{a}}$$



Dipole moment is a vector quantity having direction along dipole axis from $-q$ to $+q$. Its SI unit Coulomb meter (cm).

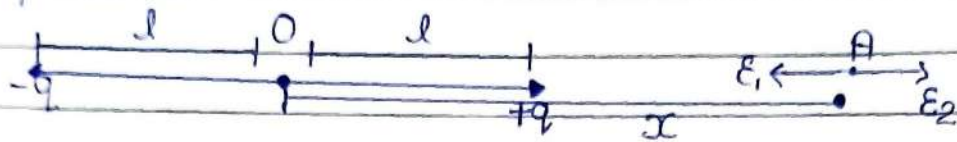
Practiced unit $\Rightarrow$ Debye.

Dipole field $\div$

The electric field produced by an electric dipole is called a dipole field.

Note $\div$ The total charge of an electric dipole is zero. But electric field of an electric dipole is not zero. This is because $+q$ and $-q$ are separated by some distance, so $\vec{E}$ field due to them when added do not exactly cancel out. However, at distance much larger than dipole size $(r \gg 2a)$ the field nearly cancel out.

Case (1) Electric field due to a dipole on axis of dipole at distance 'x' from centre of dipole :-



$$E_1 = \frac{kq}{r^2} = \frac{kq}{(x+l)^2}$$

$$E_2 = \frac{kq}{r^2} = \frac{kq}{(x-l)^2}$$

$$\text{Net electric field} = E_2 - E_1$$

$$= \frac{kq}{(x-l)^2} - \frac{kq}{(x+l)^2}$$

$$= kq \left[\frac{1}{(x-l)^2} - \frac{1}{(x+l)^2} \right]$$

$$= kq \left[\frac{(x+l)^2 - (x-l)^2}{(x-l)^3(x+l)^2} \right]$$

$$= kq \left[\frac{4lx}{(x^2-l^2)^2} \right]$$

$$= \frac{2kq(2l)x}{(x^2-l^2)^2} \quad [\because p = q \times 2l]$$

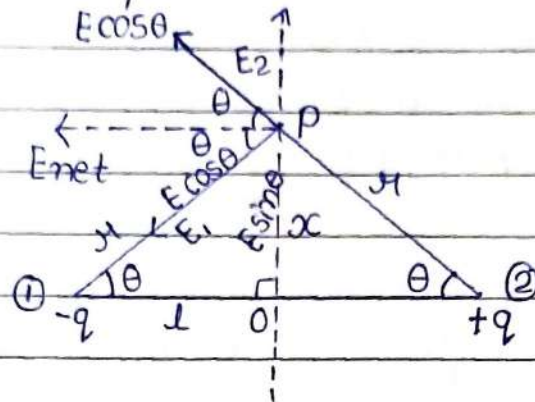
$$= \frac{2kPx}{(x^2-l^2)^2} \quad [\because x \gg l \text{ i neglect } l^2 \rightarrow 0]$$

$$= \frac{2kPx}{x^4}$$

$E_{\text{net}} = \frac{2kP}{x^3}$	$\vec{E}_{\text{axis}} = \frac{2k\vec{P}}{x^3}$
------------------------------------	---

→ $\vec{E}_{\text{net}}$ and $\vec{P}$ have same direction.

Case ② - Electric field due to a dipole on perpendicular bisector of dipole at 'x' distance from Centre of dipole.



$$r^2 = l^2 + x^2$$

$$r = \sqrt{l^2 + x^2}$$

$$\vec{E}_{-q} = \frac{kq}{(\sqrt{l^2 + x^2})^2} = \frac{kq}{(l^2 + x^2)}$$

and,

$$\vec{E}_{+q} = \frac{kq}{(l^2 + x^2)}$$

Thus, magnitude of $\vec{E}_{-q}$ and $\vec{E}_{+q}$ are equal, so, the net electric field $\vec{E}_{net}$ is equal to

$$E_{net} = 2 E \cos \theta \quad \left[\because \cos \theta = \frac{B}{H} = \frac{l}{r} \right]$$

$$E_{net} = \frac{2 kq \cdot l}{x^2 + l^2 \sqrt{x^2 + l^2}}$$

$$E_{net} = \frac{kq 2l}{(x^2 + l^2)^{3/2}}$$

$$E_{net} = \frac{kP}{(x^2 + l^2)^{3/2}} \quad \because [x \gg l ; l^2 \rightarrow 0]$$

$$E_{net} = \frac{kP}{x^3}$$

$$\boxed{\vec{E}_1 = -\frac{k\vec{P}}{x^3}}$$

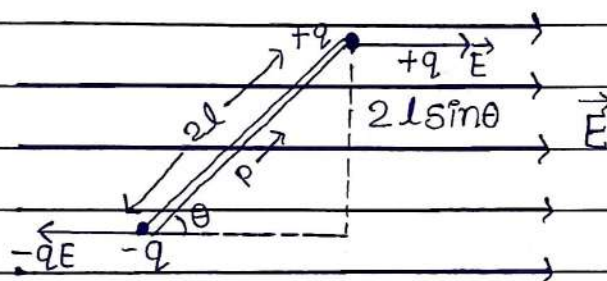
$\vec{E}_{net}$ and $\vec{P}$ is
antiparallel direction

Note → On comparing ;
clearly,

$$E_{\text{axial}} = 2 E_{\text{equatorial}}$$

Hence, at large distance dipole field falls off
not as $\frac{1}{r^2}$ but as $\frac{1}{r^3}$.

Torque on a dipole in a uniform $\vec{E}$ field:



→ Consider an electric dipole consisting of charge $+q$ and $-q$ and of length $2a$ placed in a uniform electric field $\vec{E}$ making an angle θ with it. It has dipole moment of magnitude.

$$p = q \times 2a$$

Force exerted on $+q$ by field $\vec{E} = q\vec{E}$ (along $\vec{E}$)

force " " $-q$ by " $\vec{F} = -q\vec{E}$ (opposite $\vec{E}$)

$$\vec{F}_{\text{total}} = 0$$

Net translating force on a dipole in uniform $\vec{E}$ field is zero.

But $-q$ and $+q$ form a couple which exerts a torque.

Torque = Force $\times$ $\perp$ distance between two forces.

$$\tau = qE \times 2l \sin\theta$$

$$\tau = PE \sin\theta$$

As the direction of Torque is perpendicular to both $\vec{P}$ and $\vec{E}$, so,

$$\vec{\tau} = \vec{P} \times \vec{E}$$

▷ When $\vec{P}$ is along $\vec{E}$, $\theta = 0^\circ$, $\tau = PE \sin\theta = 0$
The dipole is in stable equilibrium.

▷ When $\vec{P}$ is along $\vec{E}$, $\theta = 180^\circ$, $\tau = PE \sin 180^\circ = 0$
The dipole is in unstable equilibrium.

→ The torque will be maximum when dipole is held perpendicular to $\vec{E}$.

$$\tau_{\max} = PE \sin 90^\circ = PE$$

$$\tau_{\min} = PE \sin 0^\circ = 0$$

If $E = 1$ unit, $\theta = 90^\circ$, then $\tau = P$

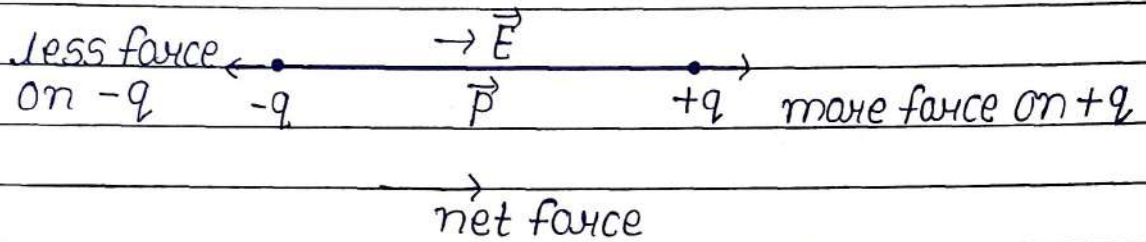
→ Dipole moment can be defined as the torque acting on an electric dipole, placed perpendicular to a uniform electric field of unit strength.

Dipole in non uniform $\vec{E}$ field :-

In either case, when $\vec{P}$ is parallel to $\vec{E}$ or anti-

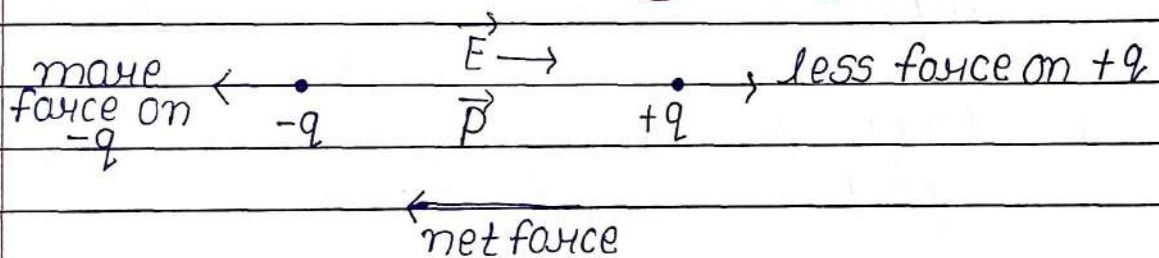
parallel to $\vec{E}$, the net force is zero, but there is a net force on dipole if $\vec{E}$ is not uniform.

(a) When field is increasing along $\vec{P}$:



Net force on dipole is along $\vec{E}$
 Net Torque = $PE \sin \theta = 0$ [$\theta = 0^\circ$]

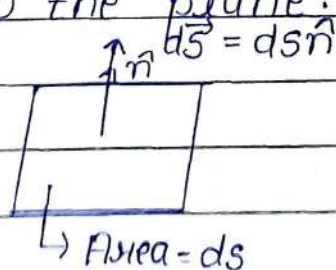
(b) When field is decreasing along $\vec{P}$:



Net force on dipole is opposite to $\vec{E}$
 Net Torque = $PE \sin \theta = 0$ [$\theta = 0^\circ$]

Area Vector :-

Area vector is a vector quantity which is acting outward perpendicular on the surface. The direction of a planar area vector is perpendicular / normal to the plane.



Electric Flux :-

The electric flux through a given area held inside an electric field is the measure of total number of electric lines of force passing normally through that area.

$$\boxed{\Phi_E = \vec{E} \cdot \vec{A} \text{ or } EA \cos \theta}$$

- Scalar quantity
- SI unit is $\text{Nm}^2\text{C}^{-1} / \text{Vm}$
- Dimension is $[M^1T^{-3}A^{-1}]$

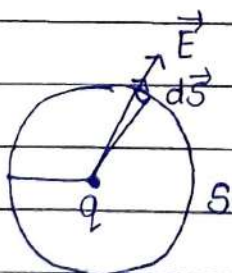
Gauss's Theorem :-

It gives a relationship between total flux passing through any closed surface and net charge enclosed by the closed surface.

Gauss theorem states that the total flux through a closed surface is $1/\epsilon_0$ times the net charge enclosed by the closed surface.

$$\boxed{\Phi_E = \oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}}$$

Proof :



Let a spherical gaussian surface S with radius r centered on q .

Electric field at any point on S is

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

Also, area element points radially outwards, so it is parallel to $\vec{E}$; $\theta = 0^\circ$

$\therefore$ Flux through area $d\vec{S}$ is

$$d\phi_E = \vec{E} \cdot d\vec{S} = E dS \cos 0^\circ = E dS$$

Total flux through surface S is

$$\phi_E = \oint_S d\phi_E$$

$$\phi_E = \oint_S E dS$$

$$\phi_E = E \oint_S dS$$

$$\phi_E = E \times \text{Total area of sphere}$$

$$\phi_E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot 4\pi r^2$$

$$\phi_E = \frac{q}{\epsilon_0}$$

This proves Gauss theorem.

Gaussian surface :-

Any hypothetical closed surface enclosing a charge is closed Gaussian surface.

Coulomb's law from Gauss Theorem :-

$\rightarrow$ Spherical gaussian surface



$\vec{E}$ and $d\vec{s}$ at any point on S are directed radially outward.

Flux through area S is —

$$\phi_E = \vec{E} \cdot d\vec{s} = Eds \cos 0^\circ = Eds$$

Net flux through closed surface S is

$$d\phi_E = \oint_S \vec{E} \cdot d\vec{s} = \oint_S Eds = E \oint_S ds$$

$$d\phi_E = E \times 4\pi r^2$$

By Gauss theorem

$$\phi_E = \frac{q}{\epsilon_0}$$

$$E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

Force on point charge q_0 if placed on surface S will be

$$F = q_0 E = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{r^2}$$

This proves the coulomb's law.

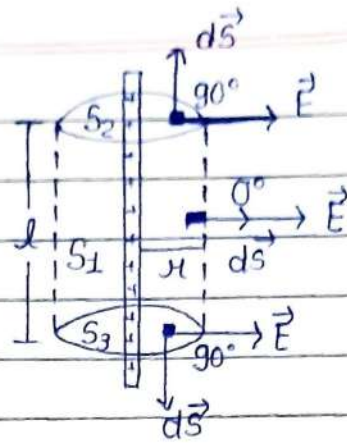
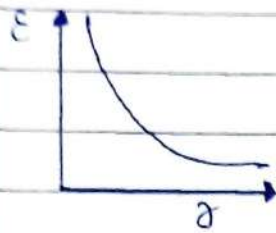
Application of Gauss theorem :-

1) Field due to an infinitely long straight uniformly

Consider a thin infinitely long straight thin wire having uniform charge density $\lambda \text{ cm}^{-1}$.

To determine the field at a distance r from the line charge, we choose a cylindrical Gaussian surface.

E vs x graph



As shown in figure it has curved surface S_1 and flat circular ends S_2 and S_3 .

$$\begin{aligned}\phi_E &= \oint_S \vec{E} \cdot d\vec{s} = \oint_{S_1} \vec{E} \cdot d\vec{s} + \oint_{S_2} \vec{E} \cdot d\vec{s} + \oint_{S_3} \vec{E} \cdot d\vec{s} \\ &= \oint E ds \cos 0^\circ + \oint E ds_2 \cos 90^\circ + \oint E ds_3 \cos 90^\circ\end{aligned}$$

$$= E \int ds_1 + 0 + 0$$

$$= E \times \text{area of curved surface}$$

$$\phi_E = E \times 2\pi r l \quad \text{--- (1)}$$

Using Gauss theorem,

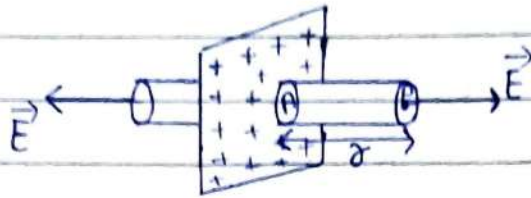
$$\textcircled{2} - \phi_E = \frac{q}{\epsilon_0}, \text{ we get}$$

$$E \cdot 2\pi r l = \frac{q}{\epsilon_0} \quad [\because q = \lambda l]$$

$$E \cdot 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$E = \frac{\lambda}{2\pi \epsilon_0 r}$

2). Electric field due to a uniformly charged infinite plane sheet.



Consider a thin plane sheet of charge with uniform surface charge density σ . we have to calculate $\vec{E}$ field from a distance x to a point P .

We choose cylindrical Gaussian surface of cross-sectional area A and length $2x$ with its axis $\perp$ to the sheet.

As lines of force are parallel to the curved surface of cylinder, the flux through the curved surface is zero.

Flux through plane end,

$$\Phi_E = EA + EA = 2EA$$

Charge enclosed by a Gaussian surface, $q = \sigma A$
According to Gauss's theorem,

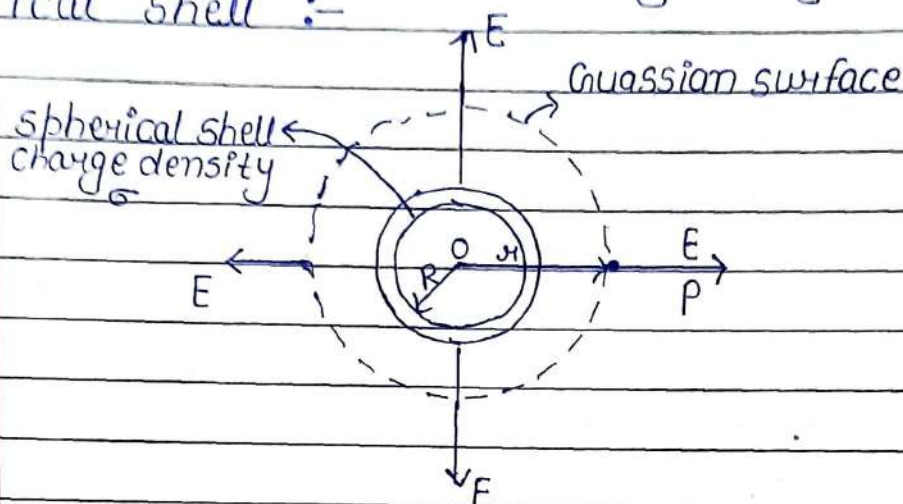
$$\Phi_E = \frac{q}{\epsilon_0}$$

$$\therefore 2EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

In vector, $\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n}$

(3) Field due to a uniformly charged thin spherical shell :-



(a) When point P lies outside the spherical shell.

To determine electric field at any point P at a distance of radius r as Gaussian surface.

Total charge q inside gaussian surface is charge on the shell of radius R and area $4\pi R^2$.

$$q = 4\pi R^2 \sigma$$

Flux through gaussian surface :

$$\phi_E = \oint \vec{E} \cdot d\vec{s}$$

$$\phi_E = E \times \text{area} \quad [\text{For } R < r]$$

$$\phi_E = E \times 4\pi r^2$$

By gauss theorem ;

$$\phi_E = \frac{q_{\text{inside}}}{\epsilon_0}$$

$$E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}} \Rightarrow \vec{E} = \frac{q}{4\pi r^2} \cdot \hat{r}$$

(b) When point P lies on spherical shell.

The gaussian surface just encloses the charged spherical shell. [$r = R$]

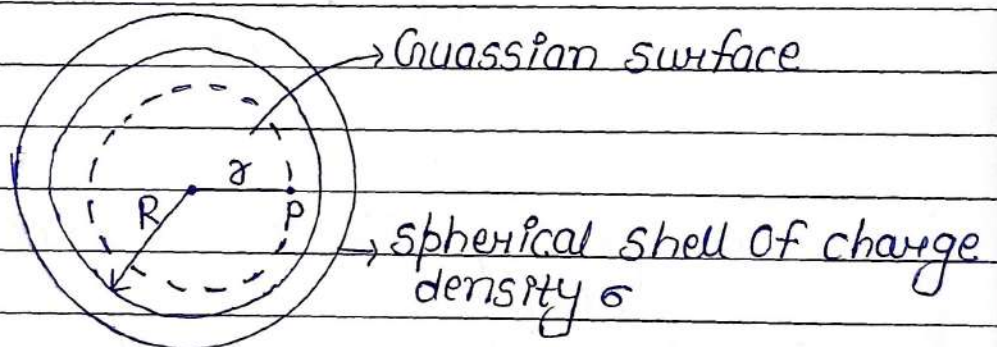
By gauss theorem;

$$E \times 4\pi R^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 R^2}$$

$$E = \frac{\sigma}{\epsilon_0} \quad \because [q = 4\pi R^2 \sigma]$$

(c) When point P lies inside the surface of spherical shell:



We know that q inside gaussian surface is zero.

$$q = 0.$$

Flux through gaussian surface

$$\phi_E = E \times 4\pi r^2$$

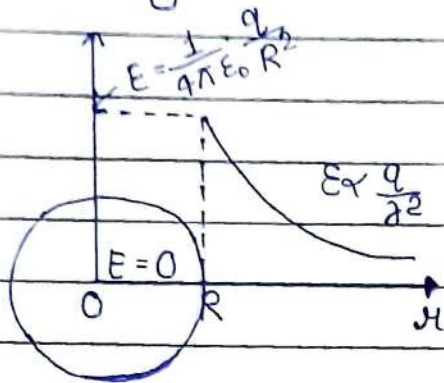
Applying gauss theorem,

$$\phi_E = \frac{q}{\epsilon_0}$$

$$E \times 4\pi r^2 = 0$$

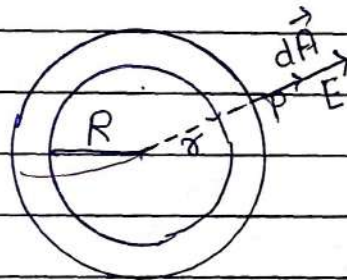
$$E = 0$$

Variation of E with x for a spherical shell of charge.



E varies with distance x from centre of shell of radius R . E is zero from $x=0$ to $x=R$.

④ Field Due to a uniformly charged insulating sphere :-



① When point p lies outside the sphere.
($x > R$)

The total charge inside the gaussian surface is the charge inside the gaussian surface is the charge inside the sphere of radius R .

$$\rho = \frac{q}{V}$$

$$q = \frac{4}{3} \pi R^3 \rho$$

Flux through the gaussian surface,
 $\phi_E = E \times 4\pi x^2$

By Gauss theorem,

$$\phi_E = \frac{q_{\text{inside}}}{\epsilon_0}$$

$$\phi_E = \frac{4/3 \pi R^3 \rho}{\epsilon_0} \quad \text{--- (2)}$$

$$\therefore E \times 4\pi r^2 = \frac{4}{3} \frac{\pi R^3 \rho}{\epsilon_0}$$

$$E \times 1^2 = \frac{R^3 \rho}{3 \times \epsilon_0}$$

$$E = \frac{\rho R^3}{3 \epsilon_0 \text{ m}^2}$$

or

$$E = \frac{q}{4\pi \epsilon_0 \text{ m}^3} \cdot \hat{r}$$

(b) When point P lies on the sphere.

The gaussian surface just encloses the charge sphere. Applying the gauss theorem,

$$\phi_E = E \times 4\pi R^2 \quad \text{--- (1)}$$

$$\text{also, } \phi_E = \frac{q_{\text{inside}}}{\epsilon_0} \quad \text{--- (2)}$$

$$E \times 4\pi R^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi R^2 \epsilon_0}$$

[for $r = R$]

$$E = \frac{q}{4\pi \epsilon_0 R^2}$$

© When point P lies inside sphere.

The charge enclosed by gaussian surface of radius r is -

$$\rho = \frac{q}{V} ; q = \frac{4}{3} \pi r^3 \rho$$

Flux through gaussian surface ;

$$\phi_E = E \times 4\pi r^2 \quad \text{--- (1)}$$

By gauss theorem ;

$$\phi_E = \frac{q}{\epsilon_0} = \frac{4/3 \pi r^3 \rho}{\epsilon_0}$$

$$\phi_E = \frac{4\pi r^3 \rho}{3\epsilon_0} \quad \text{--- (2)}$$

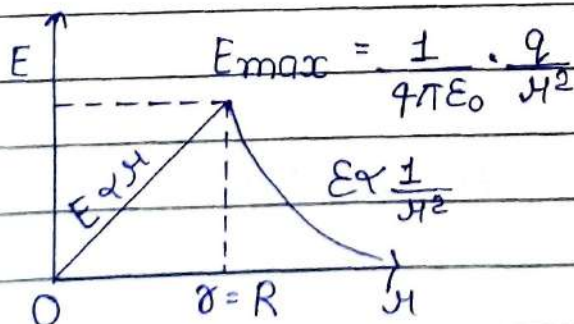
From (1) and (2) -

$$E \times 4\pi r^2 = \frac{4\pi r^3 \rho}{3\epsilon_0}$$

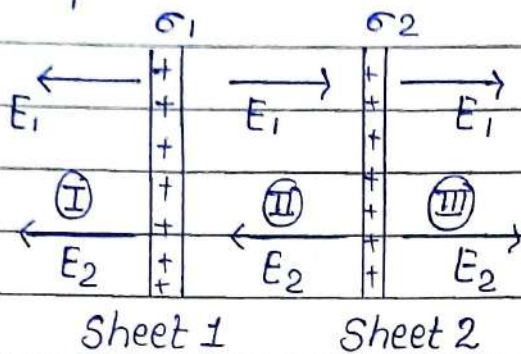
$$E = \frac{\rho r}{3\epsilon_0}$$

$$E = \frac{\rho r}{3\epsilon_0}$$

Graph :



Electric field of two positively charged parallel plates.



In region I:

$$\vec{E}_1 = \frac{-\sigma_1}{2\epsilon_0} \hat{A}, \quad \vec{E}_2 = \frac{-\sigma_2}{2\epsilon_0} \hat{A}$$

From Superposition principle:

$$\vec{E}_I = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_I = \frac{-\hat{A}}{2\epsilon_0} (\sigma_1 + \sigma_2)$$

In region II:

$$\vec{E}_1 = \frac{\sigma_1}{2\epsilon_0} \hat{A}, \quad \vec{E}_2 = \frac{-\sigma_2}{2\epsilon_0} \hat{A}$$

$$\vec{E}_{II} = \frac{\hat{A}}{2\epsilon_0} (\sigma_1 - \sigma_2)$$

In region III:

$$\vec{E}_1 = \frac{\sigma_1}{2\epsilon_0} \hat{A}, \quad \vec{E}_2 = \frac{\sigma_2}{2\epsilon_0} \hat{A}$$

$$\vec{E}_{III} = \frac{\hat{A}}{2\epsilon_0} (\sigma_1 + \sigma_2)$$